Sourcing with Deferred Payment and Inspection under Supplier Product Adulteration Risk

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We study the deferred payment and inspection mechanisms for mitigating supplier product adulteration, with endogenous procurement decision and general defect discovery process. We first derive the optimal deferred payment contract, which reveals that either entire or partial deferral can arise, depending on the moral hazard severity and the information accumulation rate. Because of the supplier's incentive to adulterate, the optimal procurement quantity under deferred payment generally is smaller than the first-best quantity. We then investigate the inspection mechanism and characterize the equilibrium. We find that under the inspection mechanism, the optimal procurement quantity is no less than the first best. A comparison between these two mechanisms shows that the deferred payment mechanism generally can outperform the inspection mechanism when either the market size is small or the profit margin is low. However, we find that these two mechanisms can also be complementary, for which we characterize a necessary condition.

Key words: quality control; deferred payment; inspection; moral hazard

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1. Introduction

When a firm outsources its manufacturing functions to suppliers located far away, the firm often has difficulty monitoring the supplier's process and product quality. Knowing the firm cannot fully monitor the process creates an environment for opportunistic behaviors, including product adulteration. Indeed, supplier product adulteration has been a common cause of product quality issues, resulting in significant losses for both firms and customers. (See Babich and Tang 2012 for detailed examples.) Hence, implementing appropriate mechanisms to align the incentives of the suppliers with their own is a pressing issue facing many firms.

Traditional mechanisms to control supplier product quality include supplier certification, inspection, and product warranty. The optimal design of these mechanisms and the implications for product quality have been investigated in the academic literature. (See Section 2 for a survey.) While these mechanisms can be useful in specific environments, they might not be effective when dealing with far-away suppliers in emerging economies. Such suppliers might act opportunistically even if they are certified. Reserving the right to pay less or to impose a penalty in the contract is often necessary (Chaudhuri 2013). Inspection has been widely used to discover product quality problems before delivery, but inspection can be inaccurate and costly. Enforcing product warranty over small, overseas suppliers is often difficult. In view of the limitations of the traditional approaches in dealing with foreign suppliers, a pioneering study, Babich and Tang (2012), proposes a useful mechanism of deferred payment to deter supplier product adulteration. Under this mechanism, some fraction of the payment is withheld initially that will be released to the supplier only if no adulteration is discovered after the product has been used over a prespecified time period. Babich and Tang (2012) analyze the optimal deferred payment mechanism and compare it with inspection, under a unit procurement quantity and the exponential discovery process. Following Babich and Tang (2012), this paper studies a buyer firm's procurement strategy in the presence of supplier product adulteration risk. We endogenize the buyer's quantity decision with the consideration of general defect discovery process and investigate the interaction of quantity decision with the design of the quality control mechanisms.

We first consider the case in which the buyer uses deferred payment to manage quality. We show that the buyer might either defer the entire payment or defer some partial payment, depending on two economic driving forces: the moral hazard severity and the information accumulation rate. The former reflects the gain to the supplier of making adulterated products, while the latter is related to how quickly defect information accumulates over time. Specifically, the entire payment should be deferred if the moral hazard problem is very severe or the accumulation of defect information is slow; otherwise, partial deferral with an initial payment at delivery is optimal. The procurement quantity acts as a "scaling" factor of the defect discovery process. That is, a larger quantity accelerates defect discovery. However, the effect of procurement quantity on the contract structure is not necessarily monotone. We show that when the hazard rate function associated with the defect discovery process is decreasing, a larger procurement quantity makes partial deferral more favorable than entire deferral. However, this finding might not hold if the hazard rate function is increasing. Furthermore, we find that under the deferred payment mechanism, the optimal procurement quantity generally is smaller than the first best — the case without supplier product adulteration risk. After the deferred payment analysis, we analyze the inspection mechanism. We derive the adulteration and inspection equilibrium, based on which we find that under the inspection mechanism, the optimal procurement quantity is no less than the first-best quantity. Finally, we compare the performance of these two mechanisms. The deferred payment mechanism generally outperforms inspection if the buyer's market size is small or the profit margin is low. However, we find that these two mechanisms can also be complementary. We identify a necessary condition for the optimality of the combined mechanism.

This study makes a few useful contributions. First, we extend the exploration of the deferred payment mechanism for its application to broader environments. The identification of the two economic driving forces deepens our understanding of the structure of the optimal deferred payment contract. Second, we consider an endogenous quantity decision. Prior literature that investigates supplier–buyer incentives on quality issues often assumes an exogenous single unit procurement setting. Our study offers novel insights about the interplay between the procurement quantity and the quality control mechanisms. Third, we compare the deferred payment and the inspection mechanisms in a general setting. The findings about their relationship can be useful in making decisions about these two mechanisms in practice.

The remainder of the paper is organized as follows. Section 2 reviews the literature. In Section 3, we describe the base model. Sections 4 and 5 analyze the optimal design of the deferred payment mechanism and the inspection mechanism, respectively. We compare their performance in Section 6 and conclude in Section 7.

2. Literature Review

Our study is related to the literature that investigates quality control in supply chains. Reyniers and Tapiero (1995) study a problem where the parts supplier can choose different levels of technologies to make the parts that lead to different levels of defects and the producer can choose the probability of inspection that is precise but costly. The supplier provides a price rebate if a defective part is discovered by inspection, as well as a warranty cost for the failure of an end product if the producer does not inspect the parts. The study shows how the penalty and liability costs influence the equilibrium technology choice and inspection probability. Lim (2001) extends Reyniers and Tapiero (1995) to an adverse selection setting in which the supplier's type, which determines the quality level, is unknown to the producer. The producer designs a mechanism that consists of a menu of inspection probability, price rebate for each part that fails inspection, and liability cost for external product failure, corresponding to the potential types of the supplier. Baiman et al. (2000) study a problem similar to Reyniers and Tapiero (1995) but assume that the producer chooses the accuracy of the inspection. Starbird (2001) applies an EOQ model and explores the optimal size of the sample to inspect and the associated reward–penalty scheme.

In these studies, only the supplier's investment or type affects the quality of the end product. Differently, Baiman et al. (2001) explore a setting in which both the supplier and the producer can influence the end product's quality. The supplier, which acts as the principal in their study, first determines its investment in the quality of the parts and then offers a contract to the producer that consists of the price and warranty. The producer determines the inspection level, as well as the quality of the production process. Baiman et al. (2001) examine the effects of product design and information structure on quality control. Balachandran and Radhakrishnan (2005) study a similar problem where the failure of the end product is determined by both the supplier's and the producer's quality levels, but in their study, the producer acts as the principal. Lee and Li (2012) explore the optimal inspection and incentive contracts in different scenarios where the quality improvement efforts exerted by the supplier and the producer can be either complementary or substitutive.

Supplier certification can also be a useful quality control mechanism. Hwang et al. (2006) compare the performance of supplier certification with inspection. They show that the buyer's use of the inspection mechanism can induce additional cost because the supplier might perform unwanted/preemptive inspection. Even though certification does not perfectly identify the supplier's quality level, it might save the moral hazard agency cost that would arise under the inspection mechanism. Chao et al. (2009) focus on the recall event and examine the quality improvement performances of two types of contracts (i.e., cost-sharing based on selective root cause analysis vs. partial cost-sharing based on complete root cause analysis) that specify the sharing of the recall cost among the parties. Wang et al. (2013) generalize this study by considering both the quantity decision and the recall risk.

Departing from the traditional literature, Babich and Tang (2012) study a new mechanism in which the buyer firm can defer payment to the supplier that will be issued only if no customer complaints about the product quality are reported up to a prespecified period of time. They discuss that this mechanism can be effective when a buyer firm trades with suppliers in emerging economies. Their analysis focuses on the project-type procurement environment with an exponential defect discovery process and finds that the entire payment is deferred at optimum. Babich and Tang (2012) also examine several factors that affect the efficiency of this mechanism, including the supplier's adulteration cost and financing cost. Our work extends their study to consider an endogenous procurement quantity and general defect discovery process.

3. The Base Model

We consider a risk-neutral buyer that plans to procure some products from a risk-neutral supplier and then sell them to the end customers. Similar to Babich and Tang (2012), the buyer faces a moral hazard problem with respect to the supplier's action. In particular, the supplier can either follow the prespecified production process, avoid adulteration, and produce non-defective products ($a = n$), or can bypass the specifications, choose adulteration, and produce defective products ($a = d$). The cost of producing a unit of unadulterated products is c_n, which is greater than the cost of producing an adulterated product c_d (i.e., $c_d < c_n$). The buyer cannot observe the supplier's action, which creates an incentive for the supplier to reduce the production cost by adulterating. We let $\theta = c_d/c_n$ to indicate the supplier's product adulteration incentive. Clearly, the smaller the θ is, the more the supplier can gain from undiscovered adulteration, and hence the more severe is the supplier's moral hazard problem.

The supplier produces the products immediately if he is contracted. The buyer decides whether to accept the products when the production is finished at time zero. If the buyer accepts them, the products are shipped to the stores and then sold to the end customers. We use $L(>0)$ to capture the lead time — the time from when the production is finished to when the products are delivered to the stores. (We do not include the production lead time because in our context the earliest time that the buyer might pay the supplier is when the production is finished and all the products are ready to be distributed. The production lead time, if modeled, would appear in a common discount factor under both quality control mechanisms, which does not play any significant role.) When the supplier uses an adulterated production process, the defects in the products will eventually be discovered by the customers and reported to the buyer. No defects are reported if the supplier follows an unadulterated production process. We assume that for any sold quantity q, the defect discovery time of each unit of the adulterated products follows: $\tau_i = L + z_i$, $i \in \{1, ..., q\}$, where the lead time L acts as a common factor and the individual z_i is a nonnegative random variable which can be either correlated with or independent of each other. z_i might contain both the shelf time before the individual product is bought by a customer and the usage time afterwards. Notice that based on the assumption that an unadulterated production process never generates any defect, the earliest report of a product defect perfectly reflects that the supplier used an adulterated process. Therefore, let $\tau = \min\{\tau_1, \cdots, \tau_q\}$, which is the first time a product defect is reported by a customer, and let $F(t) = \mathbf{P}(\tau \leq t)$ be the associated distribution function, which depends on q.

If a product is reported as defective by a customer at time τ, the buyer recalls all the products and pays a liability ρ_B to each customer at that time. Following Babich and Tang (2012), we assume that the supplier's liability is zero to capture the fact that the supplier's product liability is rarely enforceable in practice when a domestic buyer deals with small overseas suppliers in emerging economies. However, the buyer can implement the deferred payment mechanism by withholding a fraction of the total payment. The buyer has a continuously compounded financing rate $\alpha_B > 0$, while the supplier has a financing rate $\alpha_S > 0$. To focus on the nontrivial case, we assume $\alpha_S > \alpha_B$ (i.e., obtaining a loan from the capital market is more costly for the supplier).

Finally, we assume that the buyer faces a linear, downward-sloping demand function in the selling price. In particular, let r represent the present value of the selling price, and the customer demand follows:

$$q(r) = s\big(1 - br\big).$$

Clearly, $s(>0)$ captures the market size of the buyer, and b captures the price sensitivity of the demand. For convenience, we also use $r(q)$ to express the inverse function of $q(r)$. Note that this specific linear demand function is used purely for simplicity. In fact, for any fixed q, the form of the demand function does not play any role because r is simply a parameter when q is fixed. Our analysis can be generalized to other demand forms. To ensure that quality control is necessary, we assume that the present value of the liability of selling an adulterated product, $v_B = \rho_B \mathbb{E}\left[e^{-\alpha_B \tau}\right]$, is always greater than the present value of the optimal selling price that the buyer can charge, $r(q)$.

Except for the supplier's action, which is observable only to himself, all information is common knowledge. For the purpose of benchmarking, we define the first-best solution as the one in which the buyer can force the supplier to produce unadulterated products and compensate him for his total production cost. Consequently, the buyer solves: $\max_q\{r(q)q - c_n q\}$, from which the first-best quantity follows: $q^o = \frac{s\kappa}{2}$ where $\kappa = 1 - bc_n$. Note that based on our demand function, r is bounded above by $1/b$, at which the profit margin $\frac{r-c_n}{r} = 1 - bc_n$. Hence, κ represents the highest profit margin that the buyer can achieve, and we can also rewrite the demand function as $q(r) = s\big(1 - r\frac{1-\kappa}{c_n}\big)$. In the following sections, we analyze the cases in which the supplier's production decision is not directly enforceable and the buyer ensures quality using either deferred payment, inspection, or both.

4. The Deferred Payment Mechanism

In this section, we consider the case in which the buyer uses only deferred payment to ensure product quality. With this mechanism, the buyer always accepts the products from the supplier. Let $\{q, Y_0, Y_1, T\}$ denote the contract, where q is the procurement quantity, Y_0 is the initial payment to the supplier when production is finished (i.e., at time zero), and Y_1 is the deferred payment that will be released to the supplier at time T if customers do not report a defect by then. Apparently, Y_1 must be greater than zero and T must be larger than L. Therefore, the buyer's problem can be formulated as:

$$\begin{aligned} &\max_{q, Y_0 \geq 0, Y_1 > 0, T > L} \left\{r(q)q - Y_0 - Y_1 e^{-\alpha_B T}\right\} \\ s.t.\ &Y_0 + Y_1 e^{-\alpha_S T} - qc_n \geq Y_0 + Y_1 e^{-\alpha_S T}\left(1 - F(T)\right) - qc_d, \\ &Y_0 + Y_1 e^{-\alpha_S T} - qc_n \geq 0, \end{aligned}$$

where the first inequality constraint represents the supplier's incentive compatibility (IC) constraint and the second is the supplier's participation (IR) constraint. Let $\alpha = \alpha_S - \alpha_B$, the difference of the supplier's and the buyer's financing rates, and $Y = Y_1 e^{-\alpha_S T}$, the present value of the deferred payment from the supplier's perspective. We can then rewrite the buyer's problem to:

$$\begin{aligned} &\max_{q, Y_0 \geq 0, Y > 0, T > L} \left\{r(q)q - Y_0 - Y e^{\alpha T}\right\} \\ s.t.\ &YF(T) - q\left(c_n - c_d\right) \geq 0, \\ &Y_0 + Y - qc_n \geq 0. \end{aligned} \tag{1}$$

Let $\pi_B^D(q, Y_0, Y, T)$ denote the buyer's profit under the deferred payment contract $\{q, Y_0, Y, T\}$, and let $\pi_S^D(q, Y_0, Y, T)$ be the supplier's corresponding profit. Notice that solving the optimal procurement quantity is technically challenging because the defect discovery distribution $F(T)$ can depend on q in a complex manner. Hence, for the rest of this section, we first characterize the optimal contract structure with fixed q and then analyze the property of the optimal q numerically.

4.1. The Optimal Contract Structure with Fixed Procurement Quantity

Note that for any given q, the optimal solution to (1) is achieved at one of the boundaries; that is, it is either a corner solution in which both the IC and IR and the nonnegative initial payment constraints are binding, or a solution where a (non-empty) subset of the constraints is binding. From (1), we can see that a decrease of T will lead to an increase in the buyer's profit, and T appears only in the IC constraint. Thus, the following lemma holds.

Lemma 1 *At optimum, the IC constraint of (1) must be binding.*

Given Lemma 1, we obtain Proposition 1 to describe all the possible outcomes of the optimal deferred payment contract.

Proposition 1 *For any given q, the optimal solution of (1) can be one of the following three cases:*

a) $Y_0 = 0$, $Y = \frac{q(c_n - c_d)}{F(T_a)}$, $T = T_a$, *the buyer's profit is* $\pi_B^D(q, Y_0, Y, T) = qr(q) - q(c_n - c_d)\frac{e^{\alpha T_a}}{F(T_a)}$, *and the supplier's profit is* $\pi_S^D(q, Y_0, Y, T) = \frac{q(c_n - c_d)}{F(T_a)} - qc_n$, *where* $T_a = \arg\min_T \frac{e^{\alpha T}}{F(T)}$;

b) $Y_0 = 0$, $Y = qc_n$, $T = T_b$, *the buyer's profit is* $\pi_B^D(q, Y_0, Y, T) = qr(q) - qc_n e^{\alpha T_b}$, *and the supplier's profit is zero, where* T_b *solves* $F(T) = 1 - \frac{c_d}{c_n}$; *or*

c) $Y_0 = qc_n - \frac{q(c_n - c_d)}{F(T_c)}$, $Y = \frac{q(c_n - c_d)}{F(T_c)}$, $T = T_c$, *the buyer's profit is* $\pi_B^D(q, Y_0, Y, T) = qr(q) - qc_n - q(c_n - c_d)\frac{e^{\alpha T_c} - 1}{F(T_c)}$, *and the supplier's profit is zero, where* $T_c = \arg\min_T \frac{e^{\alpha T} - 1}{F(T)}$.

In particular, if $T_a \le T_b$, *then (a) is optimal; if* $T_a > T_b > T_c$, *then (b) is optimal; otherwise, (c) is optimal.*

By Lemma 1, the IC constraint always binds at optimum. Case (a) in Proposition 1 shows the optimal contract with zero initial payment (i.e., entire deferral) and relaxed IR constraint. To explain the intuition, we can note from the IC constraint of (1) that the buyer can use either the amount of the deferred payment (i.e., Y) or the deferral duration (i.e., T) to deter the supplier from product adulteration. However, both approaches are costly because the buyer always needs to fully compensate the supplier for his production cost, and deferring either more of the payment or deferring for a longer duration increases the cost. Thus, the buyer needs to strike a balance between the two options, which leads to the displayed contract. In particular, T_a is the optimized duration associated with the entire deferral ($Y = \frac{q(c_n - c_d)}{F(T_a)}$) that renders the least cost for the buyer in deterring the supplier from product adulteration. However, this less constrained, zero-initial-payment, optimal deferral contract neither guarantees that the supplier obtains zero information rent nor ensures that the supplier's IR constraint is satisfied.

In view of that, case (b) in Proposition 1 gives the boundary contract with zero initial payment where the deferral duration T_b and the associated entire deferral ($Y = qc_n$) prevent the supplier from product adulteration (i.e., the IC constraint is binding) and also guarantee that the supplier obtains zero information rent (i.e., the IR constraint is binding). Because the IC constraint always binds and the IR condition has to hold, we can infer that T_b is the *longest* deferral duration under which the supplier's IC and IR constraints both can be satisfied with zero initial payment. If T_a obtained in case (a) is longer than T_b, then the associated deferred payment in that contract is not sufficient to cover the supplier's production cost (i.e., the IR constraint is violated), in which case the optimal entire deferral contract can only be in the form of case (b). Notice that the condition $T_a > (<) T_b$ can be rewritten as $\theta = c_d/c_n > (<) 1 - F(T_a)$, which allows for a more intuitive explanation. When the supplier's moral hazard problem is very severe (i.e., $\theta < 1 - F(T_a)$), the amount of deferred payment required to deter the supplier from product adulteration for the deferral duration T_a is sufficient to assure the supplier's participation. Hence, the less constrained optimal contract in case (a) should dominate. However, when the supplier's moral hazard problem is not severe (i.e., $\theta > 1 - F(T_a)$), with a deferral duration as long as T_a, only a small deferred payment is needed to curb the supplier's adulteration incentive, which, however, would not be enough to compensate the full production cost. Hence, the buyer has to increase the deferred amount (i.e., Y_1) but can defer for a shorter time, compared to the less constrained optimal solution of case (a).

In contrast to (a) and (b), case (c) in Proposition 1 gives the optimal contract for the case with a positive initial payment where T_c is the optimized duration of the partial deferral that deters the supplier from product adulteration and also balances the two payments to compensate for his production cost (i.e., the IR constraint is binding). Notice from (1) that for this contract to be the overall optimal solution, we also need $T_a > T_b$, or equivalently $\theta > 1 - F(T_a)$ because, otherwise, the less constrained optimal contract of case (a), which does not provide any costly initial payment, would always be feasible and would dominate the other two contracts. That is, a contract with a positive initial payment can be optimal only if the buyer's moral hazard problem is not severe.

The separation of cases (b) and (c) requires an extra condition, $T_b > T_c$ as shown in Proposition 1. This condition results directly from the non-negativity constraint of Y_0, but it can be expressed in a more explicit way that gives rise to a more interesting interpretation. Before we elaborate on it, we first provide a comparison of Proposition 1 with the results in prior literature, under the exponential defect discovery process.

Corollary 1 *For any given q, let $\underline{\theta} \equiv \frac{\alpha}{q\lambda + \alpha}$ and $\overline{\theta}$ be the unique solution in $[0, 1]$ solving $\frac{1-\theta}{\theta} = \frac{q\lambda}{\alpha}\left(1 - e^{-\alpha L}\theta^{\frac{\alpha}{q\lambda}}\right)$. If the stochastic components of the discovery time z_i are i.i.d. random variables following the distribution $F_0(t) = 1 - e^{-\lambda t}$, then the optimal deferred payment contract follows the* *form of Proposition 1(a) if* $\theta \le \underline{\theta}$*, the form of Proposition 1(b) if* $\underline{\theta} < \theta \le \overline{\theta}$*, and the form of Proposition 1(c) if* $\theta > \overline{\theta}$*. Hence, providing an initial payment is optimal if and only if* $\theta > \overline{\theta}$*.*

Corollary 1 shows that when the defect discovery process is exponential, we can express the conditions in Proposition 1 fully in θ for three regions, as illustrated in Figure 1. In particular, when $\theta \le \underline{\theta}$, as in Region I of Figure 1, the supplier's moral hazard problem is very severe. Therefore, deferring all the payment is optimal for the buyer, and the supplier enjoys positive information rent, corresponding to case (a) in Proposition 1. When $\underline{\theta} < \theta \le \overline{\theta}$, as in Region II of Figure 1, the moral hazard problem is moderate and the resulting optimal contract corresponds to case (b) in Proposition 1, in which the initial payment is still zero but the supplier derives no information rent. Finally, when $\theta > \overline{\theta}$, as in Region III of Figure 1, the moral hazard problem is very mild and the resulting optimal contract corresponds to case (c) in Proposition 1, in which the initial payment is positive.

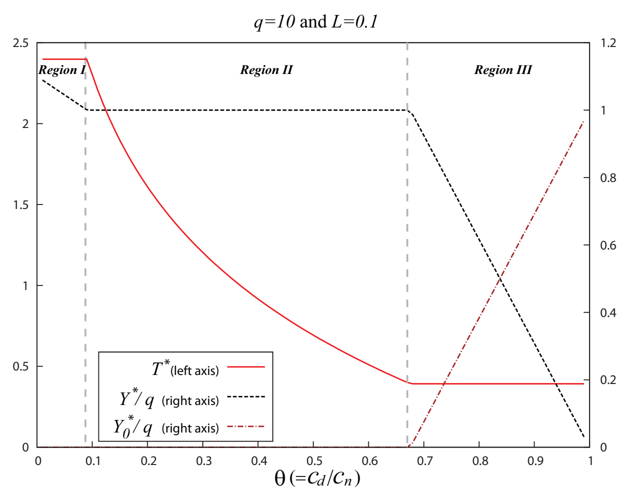

Figure 1 **Demonstration of the optimal deferred payment contract. The parameter setting:** z_i **are i.i.d. random variables following** $F_0(t) = 1 - e^{-\lambda t}$ **with** $\lambda = 0.1$, $\alpha_S = 0.2$, $\alpha_B = 0.1$, $r = 5$, $c_n = 1$, $c_d = \theta$, $L = 0.1$, $q = 10$.

To compare with the results in Babich and Tang (2012), Figure 2 provides demonstrations with $q = 1$ (left panel) and $L = 0$ (right panel). First, we can show that $\bar{\theta}$ defined in Corollary 1 decreases in q (i.e., Region III in Figure 1 shrinks as the quantity decreases). This relationship is demonstrated by the left panel of Figure 2: Here, compared to Figure 1, Region III is narrower. The procurement quantity acts as a "scaling" factor—that is, a larger selling quantity can speed up the discovery of the defect. Under the exponential distribution, the implication is that a larger quantity can mitigate the supplier's moral hazard problem and thus reduce the need to defer payment. Second, we can prove that under the exponential defect discovery process, partial deferral can be optimal (i.e., $\theta > \overline{\theta}$) if and only if $L > 0$, irrespective of the procurement quantity. In other words, when

$L = 0$, Region III completely vanishes (i.e., $\bar{\theta} = 1$), which is demonstrated by the right panel of Figure 2. Hence, Corollary 1 can fully replicate the results in Babich and Tang (2012) by setting $L = 0$ and $q = 1$. However, notice that L and q need not be large for the appearance of a significant Region III. Although the non-optimality of partial deferral appears to result fully from a zero lead time, the fundamental reason in fact has its root in the property of how the defect information probabilistically unfolds over time compared to the accumulation of the interest cost.

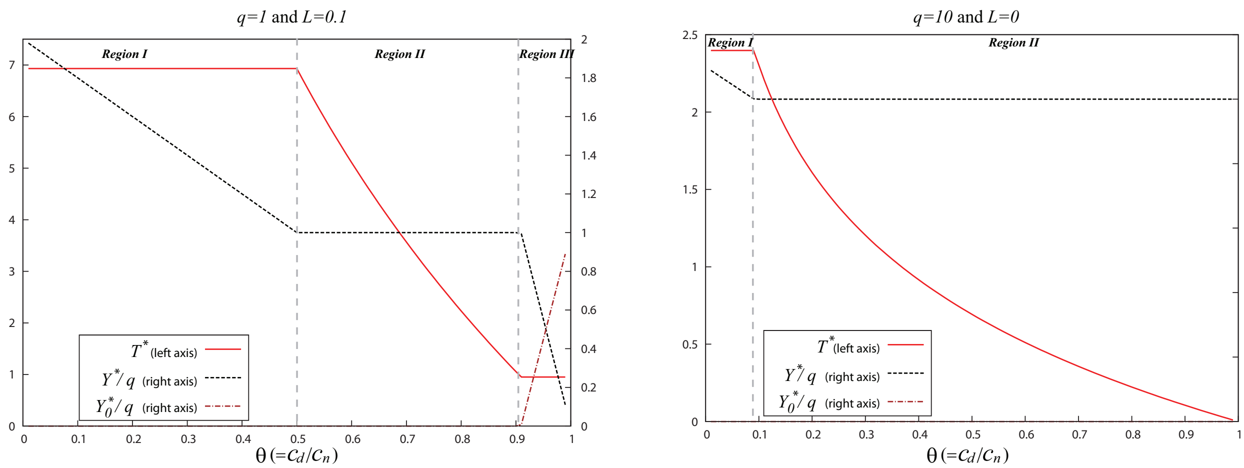

Figure 2 Demonstration of the effects of procurement quantity and lead time on the optimal deferred payment contract. The parameter setting: z_i **are i.i.d. random variables following** $F_0(t) = 1 - e^{-\lambda t}$ **with** $\lambda = 0.1$, $\alpha_S = 0.2$, $\alpha_B = 0.1$, $r = 5$, $c_n = 1$, $c_d = \theta$. **For the left panel,** $L = 0.1$, $q = 1$; **For the right panel,** $L = 0$, $q = 10$.

We now proceed to elaborate on the original conditions in Proposition 1. Although the main insights apply to general settings, we restrict some of our discussion to a family of defect discovery processes, based on which we can derive stronger results.

Definition 1. Given a discount rate $\alpha \in (0, 1)$, a cumulative distribution function $G(t)$ with the support $[0, \infty)$ is said to satisfy the *discounted unimodality* property if the function $\frac{G(t)}{e^{\alpha t} - 1}$ is unimodal.

$F(t)$ can sufficiently satisfy the discounted unimodality property if the stochastic components of the discovery time z_i are independent and are identically distributed following the common distributions such as Weibull (exponential distribution is a special case), Gompertz, Log Normal, Gamma, Pareto, etc. Hence, this property is fairly general.

Notice that for any function $J(t)$, $\frac{d}{dt}\big(\ln J(t)\big)$ measures its instantaneous growth rate. Hence, $\frac{d}{dt}\big(\ln F(t)\big)$ measures the instantaneous growth rate of the probability of detecting the defects. In other words, $\frac{d}{dt}\big(\ln F(t)\big)$ captures how fast information on supplier adulteration accumulates. On the other hand, $\frac{d}{dt}\big(\ln(e^{\alpha t} - 1)\big)$ indicates the instantaneous growth rate of the interest cost. The relative size of these two growth rates turns out to be crucial to the optimality of positive initial payment.

Corollary 2 *For any given q, partial deferral with a positive initial payment is optimal if the following two conditions are satisfied:*

- *Moral hazard condition: $\theta > 1 - F(T_a)$; that is, the moral hazard problem is not too severe;*
- *Information accumulation condition: $\frac{d}{dt}\big(\ln F(t)\big)\big|_{t=T_b} > \frac{d}{dt}\big(\ln(e^{\alpha t} - 1)\big)\big|_{t=T_b}$; that is, at the boundary deferral duration without initial payment, the instantaneous growth rate of information accumulation exceeds the instantaneous growth rate of interest cost.*

If $F(t)$ satisfies the discounted unimodality property, then these two conditions also are necessary.

The first is the moral hazard condition. Intuitively, for partial deferral to be optimal, the supplier's adulteration incentive must not be very strong. To understand the second condition, we first note that without providing any initial payment, T_b is the longest duration that the buyer can withhold the payment at optimum, as already discussed. However, if we allow some initial payment, the rest of the payment can be withheld for a longer duration, which provide more incentive for the supplier not to adulterate. Whether the latter strategy is more attractive depends on how much the chance of detecting the defects increases given the prolonged deferral duration. Clearly, the higher the probability of detecting the defects, the lower the payment the buyer needs to offer the supplier, given that his adulteration incentive will be suppressed. However, a longer deferral duration can also be expensive because the interest cost is higher for the same payment. The second condition in Corollary 2 captures this trade-off. When this condition holds at the boundary duration T_b, it suggests that the benefit of having a longer deferral duration is greater than the cost of having it. Thus, the buyer should increase the duration, which will trigger initial payment at optimum. Put simply, if the defect information is to arrive sufficiently fast at T_b, then extending the deferral duration over this time point and allocating some payment upfront can be more effective in curbing the supplier's adulteration incentive than increasing the amount of deferred payment. This condition is apparently only a sufficient one because it can be alternatively assessed at any $T > T_b$. The discounted unimodality property of $F(t)$ guarantees that these two instantaneous rates cross once, and thus this condition also becomes necessary. Interestingly, under the exponential distribution with zero lead time, the left side in the second condition never exceeds the right side at T_b (thus partial deferral cannot be optimal), while an increase in the lead time can make this condition more likely to hold.

The proceeding discussion reveals that both the quantity and the lead time can influence the contract structure. Thus, obtaining a deeper understanding of their roles would be useful. We present several structural properties in the following paragraphs.

Corollary 3 *Suppose $F(t)$ satisfies the discounted unimodality property. Then, if providing a positive initial payment is optimal under some lead time $L = L_1$, providing a positive initial payment is also optimal under $L_2 > L_1$.*

Clearly, an increase of the lead time delays the defect discovery process, which implies that the cost of deferred payment is higher. The effect on the structure of the optimal contract, however, depends on the information accumulation condition. From Corollary 2, we find that when $F(t)$ satisfies the discounted unimodal property, then once the information accumulation condition is satisfied for a certain lead time, it also will be satisfied for any longer lead time. Hence, partial deferral continues to be optimal.

Corollary 4 *Suppose $F(t)$ satisfies the discounted unimodality property and the stochastic components z_i are i.i.d. random variables following a distribution function $F_0(\cdot)$ that has a decreasing hazard rate $f_0(t)/(1-F_0(t))$. Then, if providing a positive initial payment is optimal under some procurement quantity $q = q_1$, providing a positive initial payment is also optimal under $q_2 > q_1$.*

In contrast to the lead time, an increase in the procurement quantity accelerates the defect discovery process, which can thus lower the cost to defer payment for the same deferral duration. However, the effect on the optimal contract structure is complex, depending on the information accumulation condition. To guarantee that the information accumulation condition has a monotone structural property in the procurement quantity, we need the stochastic components of the discovery process to have a decreasing hazard rate, in addition to the discounted unimodality property. In fact, if the hazard rate is increasing, then the effect of the procurement quantity on the optimal contract structure is obscure. For specific distribution functions, we find that an increase in the procurement quantity can make partial deferral either more or less preferable. Furthermore, to derive the result of this corollary, we assume the stochastic components z_i are independent. While analyzing the case with positively correlated discovery processes is challenging, we anticipate that in such a scenario $F(t)$ increases at a slower pace in q, but the structural property with respect to the optimality of a positive initial payment might be similar.

4.2. The Optimal Procurement Quantity

The previous discussion reveals how the procurement quantity might influence the optimal deferred payment contract. This effect naturally plays a role in the determination of the optimal procurement quantity. However, analytically solving the optimal quantity is challenging because $F(t)$ has a complex structure in q. We thus resort to a numerical analysis.

For convenience, we use the variant of the demand function as discussed in Section 3: $q(r) = s\big(1 - r\frac{1-\kappa}{c_n}\big)$. Recall that the first-best procurement quantity is $q^o = \frac{s\kappa}{2}$. In our experiments with supplier product adulteration incentive, we assume the stochastic components of the discovery time z_i are i.i.d. exponential random variables following the distribution $F_0(t) = 1 - e^{-\lambda t}$. Then, we numerically derive the optimal procurement quantity (q^*) and compare it with the first-best quantity. Figure

3 reports some of our results. We find that the optimal procurement quantity is generally smaller than the first best, mainly because the supplier's moral hazard problem causes an increase in the marginal procurement cost. Specifically, Figure 3 shows that the quantity difference is larger when the buyer's market size increases or when the profitability decreases. In such situations, the difference of the marginal procurement cost becomes higher. In our experiments, we also find that the optimal procurement quantity decreases as the supplier's adulteration incentive increases (i.e., θ decreases), as the difference between the two parties' financing costs increases (i.e., α increases), and as the defect discovery process slows (i.e., λ decreases). These results are intuitive because the marginal procurement cost becomes larger in such scenarios.

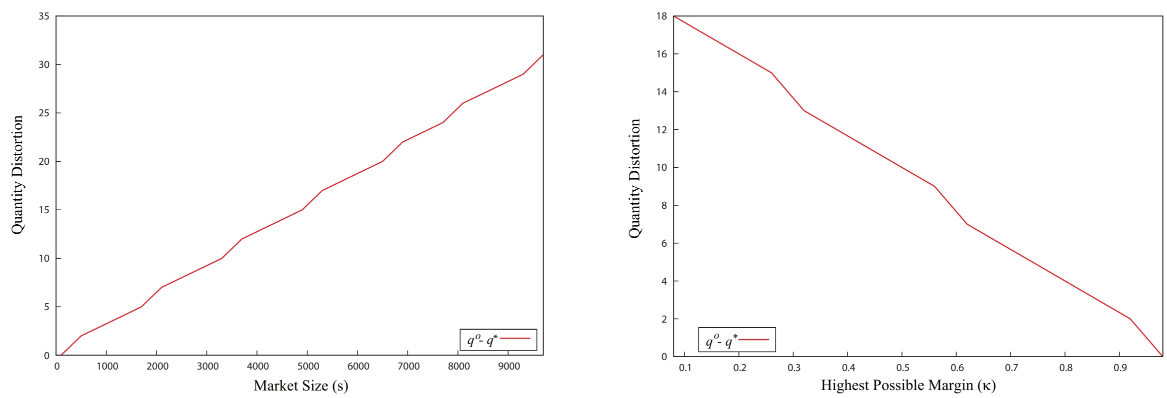

Figure 3 Demonstration of the optimal procurement quantity under the deferred payment mechanism when the buyer faces supplier product adulteration risk, compared to the first best. The largest difference is about 4% of the first best in the left plot and 9% of the first best in the right plot. The parameter setting: z_i **are i.i.d. random variables that follow the distribution** $F_0(t) = 1 - e^{-\lambda t}$ **with** $\lambda = 4$, $\alpha_S = 0.2$, $\alpha_B = 0.07$, $c_n = 1$, $c_d = 0.4$, $L = 0.1$, $\kappa = 0.2$ **(left plot), and** $s = 5000$ **(right plot).**

5. The Inspection Mechanism

In this section, we analyze the buyer's sourcing strategy when she uses inspection to discourage supplier product adulteration. Specifically, we assume that the contract contains the procurement quantity q and the payment Y_0 that is contingent on the supplier's passing inspection. After receiving the contract, the supplier makes his production decision with $a = n$ or $a = d$. We assume the production is finished instantaneously at $t = 0$, and then the buyer decides whether to inspect the products $(i = 1)$ or not to inspect them $(i = 0)$. If the buyer chooses to inspect, she decides the inspection sample size $m(\leq q)$. If the whole sample passes inspection, the buyer accepts the products and pays the supplier Y_0; otherwise, all the products are refused without any payment to the supplier. If the products are accepted, they are delivered to the buyer's stores and are ready for sale at $t = L$. Finally, the customers report to the buyer when any defect is discovered, and the buyer recalls all the products and solely incurs the liabilities, as assumed in the base model.

To conduct inspection, the buyer bears a cost $I = I_0 + mI_c$ for a sample of m units, where I_0 is the fixed cost and I_c is the variable cost. For simplicity, we assume that the inspection outcomes are independent (similar assumptions are made in the literature; see, e.g., Starbird 2001). Hence the inspection can achieve an accuracy of $\mu = 1-(1-\mu_0)^m$, where $\mu_0 = \mathbf{P}\,(\text{inspection report=adulteration}|a=d)$ is the probability that a defective product can be discovered by inspection. If the product is non-defective, then it always passes inspection (i.e., $\mathbf{P}\,(\text{inspection report=non-defective}|a=n)=1$). We confine our attention to the scenarios where the buyer can always achieve a nonnegative profit under the inspection mechanism. In equilibrium, both the buyer and the supplier may apply mixed strategies for their adulteration and inspection decisions. Let $x_S = (0,1]$ denote the supplier's adulteration probability and $x_B \in (0,1]$ be the buyer's inspection probability. Finally, let $\pi_B^I(q, Y_0, x_B, m, x_S)$ and $\pi_S^I(q, Y_0, x_B, m, x_S)$ denote the buyer's and the supplier's corresponding profit, respectively. For the rest of this section, we first characterize the inspection equilibrium for any fixed procurement quantity and then we analyze the property of the optimal quantity.

5.1. The Inspection Equilibrium with Fixed Procurement Quantity

To derive the equilibrium, we first analyze the buyer's sampling strategy when she decides to inspect the products.

Lemma 2 *For any given q, if the supplier's adulteration probability is x_S, then the optimal sampling size of the buyer's inspection strategy is as follows:*

$$m(x_S) = \min\left\{\frac{\ln\left[I_c^{-1} x_S(qr - Y_0 - qv_B)\ln(1-\mu_0)\right]}{-\ln(1-\mu_0)},\, q\right\}, \tag{2}$$

upon which the inspection accuracy is $\mu(x_S) = 1-(1-\mu_0)^{m(x_S)}$.

Lemma 2 gives the optimal sample size when the buyer decides to inspect the products. From (2), we can observe that the optimal sample size increases in the adulteration probability x_S, the quantity q, the defective loss $v_B - r$, and the initial payment Y_0 and that it decreases in the variable inspection cost I_c. More importantly, notice that, given $m(x_S)$, the inspection accuracy $\mu(x_S)$ increases in x_S; that is, if the supplier's adulteration probability x_S becomes larger, then the chance that the defective products are discovered in the inspection process also is larger, upon which the supplier will not be paid. This observation indicates the existence of an equilibrium, which we show in Proposition 2.

Proposition 2 *Let $\bar{m}$ be the unique solution of the following equation:*

$$(1-\mu_0)^{-m} + I_c^{-1}(I_0 + I_c m)\ln(1-\mu_0) = 1.$$

For any given $q > \underline{q} \equiv \frac{v_B(I_0+I_c\bar{m})}{\mu_0(c_n+v_B-r)^2}$, *the inspection game has a unique equilibrium in which the buyer conducts inspection with probability* $x_B^* = \frac{1-\theta}{1-(1-\mu_0)^{m^*}}$ *for a sample size* $m^* = \min\{\bar{m}, q\}$ *and makes a contingent payment* $Y_0^* = qc_n$ *to the supplier when the sample passes inspection; the supplier produces adulterated products with probability* $x_S^* = \frac{I_0+I_cm^*}{\left(1-(1-\mu_0)^{m^*}\right)(qc_n+qv_B-qr)}$. *The buyer achieves an equilibrium profit* $\pi_B^I(q, Y_0^*, x_B^*, m^*, x_S^*) = qr - qc_n - \frac{v_B\left(I_0+I_cm^*\right)}{\left(1-(1-\mu_0)^{m^*}\right)(c_n+v_B-r)}$, *and the supplier obtains* $\pi_S^I(q, Y_0^*, x_B^*, m^*, x_S^*) = 0$.

Note that the condition $q > \underline{q}$ is very mild, given that we focus on the quantity effect. Indeed, $\underline{q}$ is fairly small with most realistic parameters. For example, if $r = 5$, $c_n = 1$, $c_d = 0.4$, $v_B = 7.5$, $I_0 = 3$, $I_c = 0.2$, $\mu_0 = 0.6$, then $\underline{q} = 3.7$. When this condition is not satisfied, inspection equilibrium might still exist, but the analysis is much more involved.

In the equilibrium given, the buyer's inspection probability takes the ratio of $1-\theta$ to $1-(1-\mu_0)^{m^*}$. Recall that $\theta = c_d/c_n$ measures the severity of the moral hazard problem. When θ becomes smaller, the moral hazard problem is more severe, which leads to a larger inspection probability. The denominator $1-(1-\mu_0)^{m^*}$ is the inspection accuracy. When inspection becomes more accurate, the supplier's incentive to produce adulterated product is lower, which results in a smaller inspection probability. Intuitively, the optimal inspection size m^* should be larger for a smaller unit inspection accuracy μ_0. Indeed, a straightforward yet tedious derivation suggests that $\bar{m}$ is a decreasing function of μ_0. In particular, $\lim_{\mu_0\to 0} d\bar{m}(\mu_0)/d\mu_0 \to \infty$, suggesting that with very small μ_0, the optimal inspection size should be $m^* = q$.

In addition to the inspection accuracy, the supplier's adulteration probability also is influenced by the ratio of the inspection cost to the total cost the buyer incurs for selling adulterated products. In particular, when the inspection cost relative to the cost of selling adulterated products becomes larger, the supplier's incentive to produce adulterated products is higher. Note that inspection in general cannot completely deter the supplier's adulteration incentive unless the procurement quantity q goes to infinite.

Finally, having inspection with probability of 1 is possible as an equilibrium outcome. By Proposition 2, the buyer inspects with probability of 1 when $1-\theta = \mu$. In other words, when the moral hazard severity (measured by $1-\theta$) and the inspection accuracy (measured by μ) are equal, the buyer inspects for sure. Whether the inspection size m^* is equal to q depends primarily on the inspection accuracy μ_0. When μ_0 is sufficiently low, the optimal inspection size is q (i.e., 100% inspection). Notice that 100% inspection does occur in practice, especially for stringent performance requirements concerning health and safety (e.g., Mattel).

5.2. The Optimal Procurement Quantity

Based on the characterized equilibrium, we can analyze the optimal procurement quantity under the inspection mechanism. Interestingly, we find that when it satisfies the condition $q > \bar{m}$ (which is defined in Proposition 2), the optimal procurement quantity must be greater than the first best.

Proposition 3 *If the optimal procurement quantity $q^* > \bar{m}$, then $q^* > q^o$.*

Note that $\bar{m}$ is generally a small number and that $q > \bar{m}$ is a sufficient condition. Hence, the result of Proposition 3 implies that the optimal procurement quantity under inspection is greater than the first best in most cases. The intuition of this result relates to the fact that a larger procurement quantity can increase the credibility of inspection and thus reduce the supplier's adulteration probability in equilibrium. Note that, based on our numerical experiments, the difference between the optimal procurement quantity and the first best is often small (a few units). It is, however, still interesting that despite a larger procurement cost compared to the first best, the buyer orders more from the supplier if inspection is used for quality control. This finding contrasts to the finding under the deferred payment mechanism, where the optimal procurement quantity is mostly smaller than the first best.

6. Deferred Payment or Inspection

Thus far, we have characterized the optimal design of the deferred payment and inspection mechanisms. In this section, we compare their performance numerically.

6.1. Performance Comparison

To assess the performance of the two quality control mechanisms, we define the following concept:

$$\delta_j = 1 - \frac{\pi_B^j}{q(r - c_n)}, \ \forall j \in \{D, I\},$$

which measures the loss ratio of the buyer's profit under each mechanism relative to the first-best profit. A larger δ_j indicates a worse performance. In our numerical study, we first set the procurement quantity as exogenous. That is, for any given quantity, we derive the buyer's profits under the two optimized quality control mechanisms and compare them relative to the first-best profit under the same procurement quantity. This comparison can help us understand the effect of the procurement quantity and also can be useful if exogenous constraints limit the quantity choice (e.g., if the supplier has some fixed capacity constraint or production batch size constraint).

Figure 4 shows the effects of the lead time and the procurement quantity. Notice that the inspection mechanism does not depend on the lead time, while the performance of the deferred payment mechanism deteriorates as the lead time increases. Therefore, the deferred payment mechanism is generally preferred when the lead time is short. The effect of the procurement quantity is more

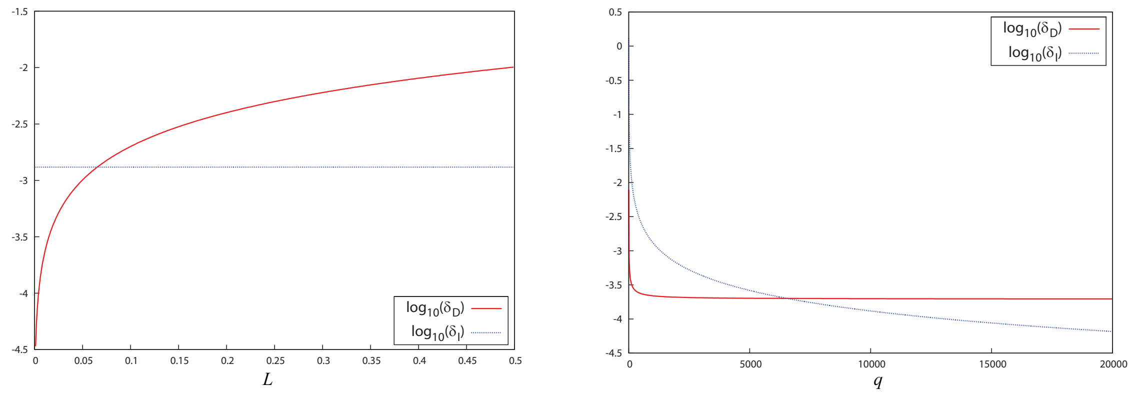

Figure 4 Demonstration of the (Log) efficiency loss ratios as functions of the lead time and the procurement quantity for the deferred payment and inspection mechanisms with exogenous procurement quantities. The parameter setting: z_i are i.i.d. random variables that follow $F_0(t) = 1 - e^{-\lambda t}$ with $\lambda = 4$, $\alpha_B = 0.07$, $\alpha_S = 0.2$, $r = 5$, $v_B = 15$, $c_n = 1$, $c_d = 0.4$, $I_0 = 3$, $I_c = 0.2$, $\mu_0 = 0.6$, $q = 1000$ (left plot), and $L = 0.01$ (right plot).

subtle. First, we can observe that the loss ratio decreases as the procurement quantity increases under either mechanism, which indicates that the performance of both quality control mechanisms improves as the procurement quantity becomes larger. Second, Figure 4 shows that the deferred payment mechanism has a better (worse) performance than inspection when q is smaller (larger) than a particular threshold. This finding indicates that the deferred payment mechanism should be chosen in scenarios where the buyer is to procure a small number of units. Third, we also have assessed the effects of the supplier's adulteration incentive (i.e., θ), the difference of the financing costs (i.e., α), the defect discovery rate (i.e., λ), and the liability cost (i.e., v_B). In general, the region where the deferred payment mechanism is preferred to inspection expands when the supplier's adulteration incentive decreases, the difference between the financing costs decreases, the defect discovery rate increases, or the liability cost decreases.

The comparison in Figure 4 assumes a given quantity. It is certainly interesting to compare these two mechanisms using endogenous quantity decision. Figure 5 reports some of our numerical findings. First, these two mechanisms lead to different optimal procurement quantities. The one under inspection generally is greater. Second, the inspection mechanism is generally preferred to the deferred payment mechanism when the buyer's market size is large or her profit margin is high. In such scenarios, the procurement quantity is large, which benefits the inspection mechanism as demonstrated in Figure 4.

6.2. Complementary or Substitutive

In addition to the performance comparison between these two mechanisms, another interesting question is whether they are complementary or substitutive when they can be jointly used. We investigate this question in this section.

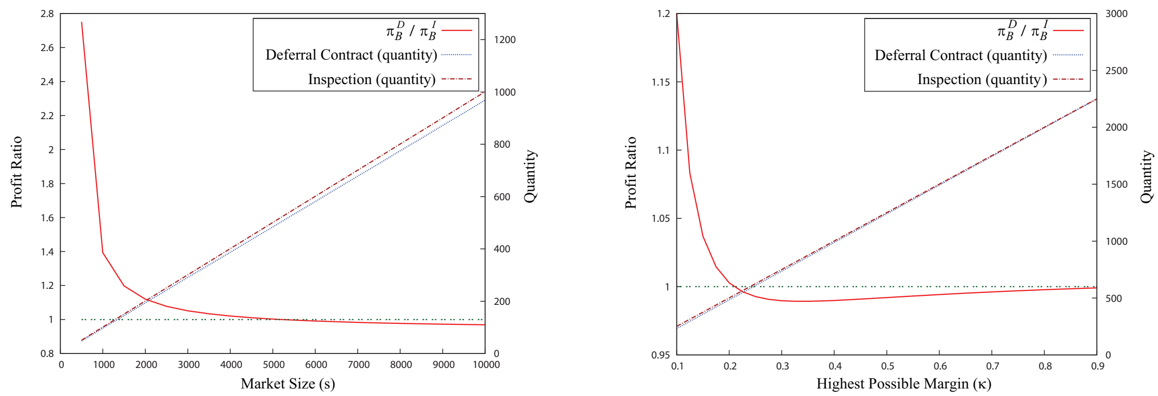

Figure 5 Comparison of the optimal procurement quantities and the profits under the deferred payment and inspection mechanisms with endogenous quantity decision. The parameter setting: z_i are i.i.d. random variables that follow the distribution $F_0(t) = 1 - e^{-\lambda t}$ with $\lambda = 4$, $\alpha_S = 0.2$, $\alpha_B = 0.07$, $c_n = 1$, $c_d = 0.4$, $L = 0.1$, $I_0 = 3$, $I_c = 0.2$, $\mu_0 = 0.6$, $\kappa = 0.2$ (left plot), and $s = 5000$ (right plot).

Apparently, when the buyer is able to make the payment term contingent on the inspection outcome, providing any non-contingent initial payment would lead to a suboptimal outcome. Therefore, we focus on the combined contract, which is a triple (Y_0, Y_1, T), where Y_0 is the payment at time zero contingent on the pass of inspection, and Y_1 is the deferred payment to the supplier that is issued at time T contingent on no defect being reported by customers by time T. Given a combined contract (Y_0, Y_1, T), the supplier chooses whether to adulterate, and the buyer simultaneously chooses whether to inspect and then sets the sample size. The buyer and the supplier again may adopt mixed strategies in equilibrium. That is, the buyer may conduct inspection with probability x_B, and the supplier may adulterate with probability x_S. We say a combined contract is nondegenerate if it is neither a deferred payment contract, described in Section 4, nor an inspection contract, described in Section 5. Clearly, a nondegenerate combined contract must have $x_B \in (0, 1]$, which also implies $x_S \in (0, 1)$ (the buyer will not participate if $x_S = 1$); moreover, it should also have $Y_1 > 0$.

Proposition 4 *Any nondegenerate combined contract with $x_B \in (0, 1)$ is suboptimal.*

We show in the proof of Proposition 4 that, given any nondegenerate combined contract (Y_0, Y_1, T), as long as the inspection probability is less than 1, we can shift a portion of the deferred payment to the initial payment, which makes both the buyer and supplier better off; and thus, the given contract is suboptimal. Therefore, Proposition 4 asserts that for a nondegenerate combined contract to be optimal, the buyer must conduct inspection with probability 1 in equilibrium (i.e., $x_B = 1$). Note that the inspection sample size does not necessarily equal the procurement quantity. In fact, we can obtain the optimal inspection size as $m^* = \min\{\bar{m}, q\}$, which has the same form as the form in the pure inspection game. Based on this preliminary result, we derive the following necessary condition for a combined mechanism to be optimal.

Proposition 5 *A nondegenerate combined contract* (Y_0, Y_1, T) *can be optimal only if*

$$1-\left[1-\left(\frac{I_0}{q}+I_c\right)\frac{v_B-r}{c_n\nu v_B}\right]^{\frac{1}{q}}<\mu_0<\min\left\{e^{-\alpha L},1-\frac{c_d}{c_n}\right\} \tag{3}$$

where

$$\nu\equiv\min\left\{(1-\theta)\frac{e^{\alpha T_a}}{F(T_a)},e^{\alpha T_b},1+(1-\theta)\frac{e^{\alpha T_c}-1}{F(T_c)}\right\}>1.$$

The subcondition $\mu_0 < e^{-\alpha L}$ in Proposition 5 is necessary to guarantee that $x_B = 1$ in equilibrium, while the subcondition $\mu_0 < 1 - \frac{c_d}{c_n}$ is necessary to ensure that the IC and IR constraints of the buyer's optimization problem can be satisfied under the given contract. The lower bound for μ_0 is intuitive. For example, when μ_0 is close to zero but the inspection cost is nonzero, then, reasonably, inspection should never be part of the optimal contract. Guided by this result, we find, interestingly, that when a non-negligible lead time exists and the procurement contract contains multiple units, the deferred payment mechanism can be either substitutive or complementary to the inspection mechanism, depending on the system parameters. In particular, we find that when inspection is not very accurate but also inexpensive, using the two mechanisms simultaneously can be beneficial. Such an example is provided in Table 1. When inspection is very accurate or very expensive, the deferred payment mechanism can either be dominated by or can dominate the inspection mechanism, and thus, these two mechanisms are substitutive in such environments. An efficient algorithm can be designed to search for the optimal combined contract for any given environment (as provided in Online Appendix).

c_n	c_d	r	v_B	q	L
0.4	0.16	1	1.5	5	0.01
I_0	I_c	μ_0	λ	α_S	α_B
0.0005	0.0001	0.1	4	0.2	0.12
Y_0^*	Y_1^*	T^*	x_S^*	m^*	δ_B^{ID}
0.47061	1.5409	0.037397	0.00063304	5	0.0029735

Table 1 **An example where the combined mechanism is optimal.** δ_B^{ID} **denotes the buyer's efficiency loss ratio under the optimal combined contract.** z_i **are i.i.d. random variables that follow** $F_0(t) = 1 - e^{-\lambda t}$**.**

Note that Babich and Tang (2012) assume $\mu_0 > 1 - \frac{c_d}{c_n}$, which provides a reasonable setting in their study. (Otherwise, the buyer would not participate in the inspection game if it is used alone.) The establishment of the necessary condition in Proposition 5 for the optimality of the combined mechanism theoretically confirms that when $\mu_0 > 1 - \frac{c_d}{c_n}$ and $L = 0$, the combined mechanism is not optimal. In particular, when the discovery time z_i follows the exponential distribution, we arrive at the conclusion of Proposition 3 in Babich and Tang (2012). Hence, the difference in the conditions of our results and those of Babich and Tang (2012) is mainly driven by our relaxed assumptions of the lead time and of the inspection accuracy with multiple procurement units.

7. Conclusion

Following the recent increased interest in quality control mechanisms to deter supplier product adulteration, we study the optimal design of the deferred payment and inspection mechanisms with endogenous quantity decision and general defect discovery process. For the deferred payment mechanism, we show that either entire or partial deferral can be optimal when the buyer procures multiple units. We identify two economic driving forces that determine the structure of the optimal deferred payment contract: the moral hazard severity and the information accumulation rate. Furthermore, we find that because of the increased procurement cost in the presence of the supplier's adulteration incentive, the optimal procurement quantity is generally smaller than the first-best quantity under the deferred payment mechanism. For the inspection mechanism, we derive the equilibrium of the supplier's adulteration decision and the buyer's inspection decision. We find that a larger procurement quantity reduces the supplier's adulteration probability, which leads to the finding that the optimal procurement quantity under inspection generally is larger than the first-best quantity. We also compare the performance of these two mechanisms. We show that the deferred payment mechanism generally is preferred to inspection when the procurement quantity and the lead time are small. This finding implies that when the buyer's market size is small, the profit margin is low, or the delivery of the products is fast, she should choose the deferred payment mechanism to deter supplier adulteration; otherwise, inspection should be used. Finally, we find that these two mechanisms can also be complementary in specific environments when they can be jointly used, and we characterize a necessary condition for the optimality of the combined mechanism. These results enrich the understandings of these two quality control mechanisms and can be useful for their implementation.

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Online Appendices for Sourcing with Deferred Payment and Inspection under Supplier Product Adulteration Risk

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Online Appendix A: Proofs

Proof of Lemma 1: This lemma is straightforward given T appears only in the IC constraint. The buyer can always reduce T to increase profit until the IC constraint binds. ■

Proof of Proposition 1: This proposition follows intuitively from the fact that the optimal solution must be achieved at one of the boundaries. Since the IC constraint must bind at optimum and we have $Y > 0$ and $T > L$, we are left with only three possibilities:

- The nonnegative initial payment constraint ($Y_0 \geq 0$) binds, the IR constraint does not bind, and the first order condition is satisfied, which leads to the result of Proposition 1(a).
- Both the nonnegative initial payment constraint ($Y_0 \geq 0$) and the IR constraint bind, which leads to the result of Proposition 1(b).
- The nonnegative initial payment constraint ($Y_0 \geq 0$) does not bind and the first order condition is satisfied, which leads to the result of Proposition 1(c). Note that IR constraint has to bind when $Y_0 > 0$. ■

The key is to recognize that the condition $T_a \leq T_b$ is a necessary condition for case (a) to be feasible (hence optimal), and is also a sufficient condition for case (a) to be optimal. Clearly, for the (IR) constraint to be satisfied in case (a), we need

$$Y_0 + \frac{q(c_n - c_d)}{F(T_a)} \geq qc_n \Leftrightarrow F(T_a) \leq 1 - \frac{c_d}{c_n} = F(T_b) \Leftrightarrow T_a \leq T_b.$$

Now suppose $T_a \leq T_b$, we know case (a) is feasible and we need to show it (weakly) dominates both case (b) and case (c).

Case (a) dominates case (b) if and only if

$$qr - q(c_n - c_d)\frac{e^{\alpha T_a}}{F(T_a)} \geq qr - qc_n e^{\alpha T_b} \Leftrightarrow \frac{c_n e^{\alpha T_b}}{c_n - c_d} \geq \frac{e^{\alpha T_a}}{F(T_a)} \Leftrightarrow \frac{e^{\alpha T_b}}{F(T_b)} \geq \frac{e^{\alpha T_a}}{F(T_a)}.$$

The last inequality is true by definition of T_a.

To compare case (a) and case (c), first notice that case (c) is feasible if and only if $T_c \geq T_b$. Indeed, if $T_c < T_b$, nonnegative constraint in case (c) will be violated:

$$Y_0 = qc_n - \frac{q(c_n - c_d)}{F(T_c)} < qc_n - \frac{q(c_n - c_d)}{F(T_b)} = 0.$$

Case (a) dominates case (c) if and only if

$$\begin{aligned} qr - q(c_n - c_d)\frac{e^{\alpha T_a}}{F(T_a)} &\geq qr - qc_n - q(c_n - c_d)\frac{e^{\alpha T_c} - 1}{F(T_c)} \\ \Leftrightarrow (c_n - c_d)\frac{e^{\alpha T_a}}{F(T_a)} &\leq c_n + (c_n - c_d)\frac{e^{\alpha T_c} - 1}{F(T_c)} \\ \Leftrightarrow F(T_b)\frac{e^{\alpha T_a}}{F(T_a)} &\leq 1 + F(T_b)\frac{e^{\alpha T_c} - 1}{F(T_c)} \\ \Leftrightarrow \frac{e^{\alpha T_a}}{F(T_a)} &\leq \frac{1}{F(T_b)} - \frac{1}{F(T_c)} + \frac{e^{\alpha T_c}}{F(T_c)} \end{aligned}$$

The last inequality is true by definition of T_a and the fact that $T_c \geq T_b$.

The only part left now is to compare case (b) and case (c). For case (c) to be optimal, it has to be feasible, that is, $T_c \geq T_b$, in which case, by definition of T_c, the profit of case (c) is higher than that of case (b). Therefore, case (b) is optimal if and only if $T_a > T_b > T_c$. ■

Proof of Corollary 1: With the exponential distribution $F_0(t) = 1 - e^{-\lambda t}$, we have $F(T) = 1 - e^{-q\lambda(T-L)}$ and $f(T) = q\lambda e^{-q\lambda(T-L)}$. We can derive:

$$T_a = L + \frac{1}{q\lambda}\ln(1 + \frac{q\lambda}{\alpha}) \text{ and } T_b = L - \frac{1}{q\lambda}\ln(\theta),$$

where $\theta = \frac{c_d}{c_n}$ as defined. Hence, the condition $T_a < T_b$ is equivalent to $\theta < \underline{\theta} \equiv \frac{\alpha}{q\lambda + \alpha}$.

Further, we can determine that T_c solves the following equation:

$$\frac{e^{q\lambda(T-L)}}{q\lambda} + \frac{e^{-\alpha T}}{\alpha} = \frac{1}{q\lambda} + \frac{1}{\alpha}. \tag{1}$$

Equation (1) has a unique solution in the interval (L, ∞) because the left-hand-side (LHS) of (1) is less than the right-hand-side (RHS) of (1) when $T = L$ and is larger than the RHS for large enough T. For case (c) to be optimal, we need $T_c \geq T_b$. Given that the LHS of (1) is increasing in T, the condition $T_c \geq T_b$ is equivalent to

$$\frac{e^{q\lambda(T_b-L)}}{q\lambda} + \frac{e^{-\alpha T_b}}{\alpha} \leq \frac{1}{q\lambda} + \frac{1}{\alpha}.$$

Substituting the formula for T_b under exponential distribution, the above inequality becomes

$$\frac{1}{q\lambda\theta} + \frac{1}{\alpha}e^{-\alpha L}\theta^{\frac{\alpha}{q\lambda}} \leq \frac{1}{q\lambda} + \frac{1}{\alpha}$$

It is easy to verify that the LHS of the above inequality is decreasing in θ. Hence, $T_c \geq T_b$ if and only if $\theta > \bar{\theta}$ where $\bar{\theta}$ is the solution to

$$\frac{1}{q\lambda\theta} + \frac{1}{\alpha}e^{-\alpha L}\theta^{\frac{\alpha}{q\lambda}} = \frac{1}{q\lambda} + \frac{1}{\alpha},$$

or equivalently,

$$\frac{1-\theta}{\theta} = \frac{q\lambda}{\alpha}\left(1 - e^{-\alpha L}\theta^{\frac{\alpha}{q\lambda}}\right),$$

which completes the proof. ■

Proof of Corollary 2: The requirement of the moral hazard condition is obvious. By Proposition 1,

$$T_a > T_b \Leftrightarrow F(T_a) > F(T_b) \Leftrightarrow \theta \equiv \frac{c_d}{c_n} > 1 - F(T_a).$$

By Proposition 1, with the moral hazard condition satisfied, only case (b) or case (c) in Proposition 1 can be optimal. We now show that the information accumulation condition will guarantee that zero initial payment is not optimal. First, notice that for both case (b) and case (c) in Proposition 1, the IC and IR constraints are both binding. Thus, we have $Y_0 = qc_n - Y$ from the IR constraint, and the objective of the buyer's problem can be written as: $\pi_B^D(q, Y_0, Y, T) = qr - Y_0 - Ye^{\alpha T} = qr - qc_n + Y\left(1 - e^{\alpha T}\right)$. Given the IC constraint binds, we have $Y = \frac{q(c_n - c_d)}{F(T)}$. Hence, the buyer's profit can be rewritten as a function of deferral time T:

$$\pi_B^D(T) = qr - qc_n + \frac{q(c_n - c_d)(1 - e^{\alpha T})}{F(T)}$$

We can then obtain the first derivative of $\pi_B^D(T)$ with respect to T:

$$\frac{d\pi_B^D(T)}{dT} = \frac{q\left(c_n - c_d\right)\left(e^{\alpha T} - 1\right)}{F(T)}\left[\frac{f(T)}{F(T)} - \frac{\alpha}{1 - e^{-\alpha T}}\right].$$

Note that as long as the deferral duration T is larger than T_b, a positive initial payment is feasible (i.e., $Y_0 > 0$). Given the moral hazard condition is satisfied, a positive initial payment is optimal if there exists a deferral duration $t > T_b$ which yields higher expected profit for the buyer.

Because

$$\left.\frac{d\pi_B^D(T)}{dT}\right|_{t=T_b} > 0 \Leftrightarrow \left.\frac{d}{dt}\Bigg(\ln F(t)\Bigg)\right|_{t=T_b} > \left.\frac{d}{dt}\Bigg(\ln(e^{\alpha t} - 1)\Bigg)\right|_{t=T_b},$$

if the information accumulation condition is satisfied, case (b) cannot be optimal. Hence, case (c) must be optimal and there is positive initial payment in the optimal contract.

We now show that with the additional assumption of discounted unimodal property, the information accumulation condition is also necessary. To see this, first notice that the discounted unimodal property implies that $\ln(F(t)/(e^{\alpha t} - 1))$ is also unimodal. Hence, the following equation has at most one solution:

$$\frac{d}{dt}\ln\left(\frac{F(t)}{e^{\alpha t} - 1}\right) = 0.$$

Rewriting the above equation, we see the following equation has at most one solution:

$$I(t) \equiv \frac{f(t)}{F(t)} - \frac{\alpha}{1 - e^{-\alpha t}} = 0 \tag{2}$$

Given the moral hazard condition is satisfied, a positive initial payment is optimal only if there exists a deferral duration $t > T_b$ which yields higher expected profit for the buyer. Clearly, this requires $\frac{d\pi_B^D(T)}{dT}$ be positive for some range of the interval $[T_b, \infty)$, or equivalently $I(t) > 0$ for some range of the interval $[T_b, \infty)$. Because $\lim_{t\to\infty} I(t) < 0$ and $I(t) = 0$ has at most one solution, a positive initial payment is optimal only if

$$I(T_b) > 0 \Leftrightarrow \frac{d}{dt}\Big(\ln F(t)\Big)\Big|_{t=T_b} > \frac{d}{dt}\Big(\ln(e^{\alpha t} - 1)\Big)\Big|_{t=T_b}.$$

■

Proof of Corollary 3: According to Corollary 2, with $F(t)$ satisfying the discounted unimodality property, if a positive initial payment is optimal when L equals some L_1, both the moral hazard condition and the information accumulation condition must be satisfied at $L = L_1$. Apparently, a change of L does not affect the moral hazard condition. Similarly, a change of L doest not affect the value of $\frac{d}{dt}\big(\ln F(t)\big)\big|_{t=T_b}$. On the other hand, $\frac{d}{dt}\big(\ln(e^{\alpha t} - 1)\big)$ is decreasing in t. Since T_b is increasing in L, the information accumulation condition will be satisfied for any $L > L_1$. This completes the proof. ■

Proof of Corollary 4: Suppose a positive initial payment is optimal when $q = q_1$. By Corollary 2, with $F(t)$ satisfying the discounted unimodality property, both the moral hazard condition and the information accumulation condition must be satisfied at $q = q_1$. Again by Corollary 2, we only need to check whether the moral hazard condition and the information accumulation condition both hold at any $q = q_2 > q_1$. Denote the hazard rate function by $H_0(t) = f_0(t)/(1 - F_0(t))$. We first check the information accumulation condition. Because

$$F(t) = 1 - \big(1 - F_0(t - L)\big)^q, \; f(t) = q\big(1 - F_0(t - L)\big)^{q-1} f_0(t - L),$$

and $F(T_b) = 1 - \theta$, we have

$$\frac{f(T_b)}{F(T_b)} = \frac{q\theta}{1-\theta} \cdot \frac{f_0(T_b - L)}{1 - F_0(T_b - L)}.$$

Define the function $g(q)$ as

$$g(q) \equiv \frac{q\theta}{1-\theta} H_0(T_b - L) - \frac{\alpha}{1 - e^{-\alpha T_b}}.$$

Because the information accumulation condition is satisfied with $q = q_1$, we have $g(q_1) \geq 0$. Hence, it suffices to show that $g(q)$ is increasing in q when $g(q) > 0$, that is,

$$\frac{dg}{dq} = \frac{\theta}{1-\theta} H_0(T_b - L) + \frac{q\theta}{1-\theta} \cdot H_0'(T_b - L) \cdot \frac{dT_b}{dq} + \frac{\alpha^2 e^{-\alpha T_b}}{(1 - e^{-\alpha T_b})^2} \cdot \frac{dT_b}{dq} > 0$$

for any q such that $g(q) > 0$.

From the equality $(1 - F_0(T_b - L))^q = \theta$, we can derive

$$\frac{dT_b}{dq} = \frac{\ln(1 - F_0(T_b - L))}{qH_0(T_b - L)} = \frac{\ln \theta}{q^2 H_0(T_b - L)} < 0.$$

Because

$$\frac{q\theta}{1-\theta} \cdot H_0'(T_b - L) \cdot \frac{dT_b}{dq} \geq 0,$$

it suffices to show that

$$\begin{aligned} \frac{\theta}{1-\theta} H_0(T_b - L) &> -\frac{\alpha^2 e^{-\alpha T_b}}{(1 - e^{-\alpha T_b})^2} \cdot \frac{dT_b}{dq} \\ \Leftrightarrow \frac{\theta}{1-\theta} H_0(T_b - L) &> -\frac{\alpha^2 e^{-\alpha T_b}}{(1 - e^{-\alpha T_b})^2} \cdot \frac{\ln \theta}{q^2 H_0(T_b - L)} \\ \Leftrightarrow \frac{1-\theta}{\theta} \cdot \left(\frac{q\theta}{1-\theta} H_0(T_b - L) \right)^2 &> \left(\frac{\alpha}{1 - e^{-\alpha T_b}} \right)^2 \cdot e^{-\alpha T_b} \cdot (-\ln \theta). \end{aligned}$$

The last inequality is true because $g(q) > 0$, $e^{-\alpha T_b} < 1$, and

$$\frac{1-\theta}{\theta} > -\ln \theta, \forall \theta \in (0, 1).$$

Second, we need to check the moral hazard condition. Denote the value of $T_a - L$, the density function, the distribution, and the hazard rate function corresponding to quantity q_i, $(i = 1, 2)$, by t_i, $f_i(t)$, $F_i(t)$, and $H_i(t)$, respectively. Apparently, $H_i(t) = q_i H_0(t)$ is also non-increasing in t.

By definition of T_a, we have

$$\frac{f_1(t_1)}{F_1(t_1)} = \frac{q_1(1 - F_0(t_1))^{q_1} H_0(t_1)}{1 - (1 - F_0(t_1))^{q_1}} = \alpha, \; \frac{f_2(t_2)}{F_2(t_2)} = \frac{q_2(1 - F_0(t_2))^{q_2} H_0(t_2)}{1 - (1 - F_0(t_2))^{q_2}} = \alpha.$$

Since the function $\psi(q) = q/(\gamma^q - 1)$ is decreasing in q for $\gamma > 1$, we have

$$\frac{q_1(1 - F_0(t_1))^{q_1} H_0(t_1)}{1 - (1 - F_0(t_1))^{q_1}} > \frac{q_2(1 - F_0(t_1))^{q_2} H_0(t_1)}{1 - (1 - F_0(t_1))^{q_2}}.$$

If $t_2 > t_1$, then because the function $\zeta(t) = \frac{(1-F_0(t))^q H_0(t)}{1-(1-F_0(t))^q}$ is decreasing in t, we have

$$\frac{q_2(1 - F_0(t_1))^{q_2} H_0(t_1)}{1 - (1 - F_0(t_1))^{q_2}} > \frac{q_2(1 - F_0(t_2))^{q_2} H_0(t_2)}{1 - (1 - F_0(t_2))^{q_2}},$$

which means

$$\alpha = \frac{q_1(1 - F_0(t_1))^{q_1} H_0(t_1)}{1 - (1 - F_0(t_1))^{q_1}} > \frac{q_2(1 - F_0(t_2))^{q_2} H_0(t_2)}{1 - (1 - F_0(t_2))^{q_2}} = \alpha.$$

Contradiction. Hence, we must have $t_2 \leq t_1$, that is, T_a is decreasing in q. Therefore,

$$H_2(t_2) \geq H_1(t_2) \geq H_1(t_1).$$

If we write $H_1(t_1)$ and $H_2(t_2)$ as

$$H_1(t_1) = \left(\frac{1}{f_1(t_1)} - \frac{1}{\alpha} \right)^{-1}, \; H_2(t_2) = \left(\frac{1}{f_2(t_2)} - \frac{1}{\alpha} \right)^{-1}$$

respectively, the inequality $H_2(t_2) \geq H_1(t_1)$ clearly implies $f_2(t_2) \geq f_1(t_1)$, or equivalently, $F_2(t_2) \geq F_1(t_1)$. This immediately implies that the moral hazard condition is satisfied with $q = q_2$. ■

Proof of Lemma 2: Given the probability of product adulteration x_S, the buyer's problem can be formulated as follows:

$$\max_m \ (1 - x_S)(qr - Y_0) + x_S(1 - \mu_0)^m(qr - Y_0 - qv_B) - (I_0 + mI_c).$$

The first order condition is $x_S(1-\mu_0)^m \ln(1-\mu_0)(qr-Y_0-qv_B)-I_c = 0$, from which we immediately have the expression for $m(x_S)$. The other claims are obvious. ■

Proof of Proposition 2: In an equilibrium, the supplier clearly has to use a mixed strategy in the second stage. Hence, we have the following indifference condition for the supplier:

$$Y_0 - qc_n = (1 - x_B)Y_0 + x_B(1 - \mu)Y_0 - qc_d,$$

which implies

$$x_B^* = \frac{q(c_n - c_d)}{\mu Y_0}.$$

If $x_B^* < 1$, i.e., $Y_0 > \frac{q(c_n-c_d)}{\mu^*}$, then the buyer is also playing a mixed strategy, which implies the following indifference condition for the buyer:

$$x_S(qr - qv_B - Y_0) + (1 - x_S)(qr - Y_0) = x_S(1 - \mu^*)(qr - qv_B - Y_0) + (1 - x_S)(qr - Y_0) - I^*,$$

or equivalently,

$$x_S^* = \frac{I^*}{\mu^*(Y_0 + qv_B - qr)}.$$

On the other hand, if $x_B^* = 1$, i.e., $Y_0 = \frac{q(c_n-c_d)}{\mu^*}$, then we must have

$$1 \geq x_S^* \geq \frac{I^*}{\mu^*(Y_0 + qv_B - qr)}.$$

Because the buyer's profit decreases in x_S, we can focus on the payoff dominant equilibrium with $x_S^* = \frac{I^*}{\mu^*(Y_0+qv_B-qr)}$, which means we can combine the two cases. The equilibrium profit for the buyer is the same as the expected profit of the buyer when she chooses inspection, i.e.,

$$\pi_B^I = x_S^*(1 - \mu^*)(qr - Y_0 - qv_B) + (1 - x_S^*)(qr - Y_0) - I^*$$

which can be simplified to

$$\pi_B^I = qr - Y_0 - \frac{qI^*v_B}{\mu^*(Y_0 + qv_B - qr)}.$$

Note that we have

$$I^* = I_0 + I_c m^* \text{ and } \mu^* = 1 - (1 - \mu_0)^{m^*},$$

where m^* satisfies

$$m^* = \min\left\{\frac{\ln\left[I_c^{-1}x_S^*(qr - Y_0 - qv_B)\ln(1-\mu_0)\right]}{-\ln(1-\mu_0)}, q\right\} = \min\left\{\frac{\ln\left[-I_c^{-1}\frac{I_0+I_c m^*}{(1-(1-\mu_0)^{m^*})}\ln(1-\mu_0)\right]}{-\ln(1-\mu_0)}, q\right\}.$$

Denote the solution of the following equation by $\bar{m}$:

$$-m\ln(1-\mu_0) = \ln\left[-I_c^{-1}\frac{I_0 + I_c m}{1-(1-\mu_0)^m}\ln(1-\mu_0)\right],$$

which could be rewritten as

$$(1-\mu_0)^{-m} + I_c^{-1}(I_0 + I_c m)\ln(1-\mu_0) = 1.$$

Denote $\zeta(m) = (1-\mu_0)^{-m} + I_c^{-1}(I_0 + I_c m)\ln(1-\mu_0)$. We have

$$\zeta'(m) = \left(1-(1-\mu_0)^{-m}\right)\ln(1-\mu_0) > 0,$$

$$\zeta''(m) = (1-\mu_0)^{-m}\ln^2(1-\mu_0) > 0.$$

which implies that $\zeta(m)$ is unbounded. On the other hand, $\zeta(0) < 1$; hence, $\bar{m}$ uniquely exists. Correspondingly, we denote $\bar{I} = I_0 + I_c\bar{m}$ and $\bar{\mu} = 1-(1-\mu_0)^{\bar{m}}$. Then, the optimal inspection size m^* is either $\bar{m}$, in which case it is independent of q, or $m^* = q$ if $\bar{m} > q$. This means m^*, I^*, and μ^* are independent of q when q is sufficiently large.

Because the supplier plays a mixed strategy, his payoff with or without adulteration is the same. Hence, the participation constraint for the supplier is

$$Y_0 - qc_n \geq 0.$$

Thus, the buyer's problem can be formulated as:

$$\begin{aligned}&\max_{Y_0}\ qr - Y_0 - \frac{qv_B I^*}{\mu^*(Y_0 + qv_B - qr)}\\ &s.t.\ Y_0 \geq qc_n.\end{aligned}$$

Under the assumption $q \geq \underline{q}$, the objective function is non-increasing in Y_0 for $Y_0 \geq qc_n$. To see this, note that the objective function is non-increasing if and only if

$$-1 + \frac{qv_B I^*}{\mu^*(Y_0 + qv_B - qr)^2} \leq 0,$$

which is equivalent to

$$\mu^*(Y_0 + qv_B - qr)^2 \geq qv_B I^*.$$

Because $Y_0 \geq qc_n$, $\mu^* \geq \mu_0$, $q \geq \underline{q}$, and $I_0 + I_c\overline{m} \geq I^*$, we have

$$\mu^*(Y_0 + qv_B - qr)^2 \geq \mu_0 q^2(c_n + v_B - r)^2 \geq qv_B I^*.$$

Therefore, the optimal solution is always achieved at the boundary; that is, $Y_0^* = qc_n$, and the buyer's corresponding profit is: $\pi_B^* = qr - qc_n - \frac{I^* v_B}{\mu^*(c_n+v_B-r)}$. Note that when $q < \underline{q}$, an interior solution may arise. The analysis would however become much more involved, which we leave for future exploration. On the other hand, the condition $q \geq \underline{q}$ is not restrictive as we explained in the main text. $\underline{q}$ is generally small for most realistic parameters. ■

Proof of Proposition 3: We rewrite the demand function as $r = \frac{s-q}{bs}$, which implies $\frac{dr}{dq} = -\frac{1}{bs}$. Notice that when $q > \overline{m}$, $m^* = \overline{m}$ which does not depend on q. Hence,

$$\frac{d\pi_B^I(q, Y_0^*, x_B^*, m^*, x_S^*)}{dq} = \frac{s - bsc_n - 2q}{bs} + \frac{v_B(I_0 + I_c m^*)}{1-(1-\mu_0)^{m^*}} \cdot \frac{1}{bs(v_B + c_n - r)^2}.$$

Clearly, at optimum, we must have $\frac{s-bsc_n-2q^*}{bs} < 0$ which immediately implies $q^* > q^o$. ■

Proof of Proposition 4: For convenience, let $R = qr$, $w = qv_B - qr$, and $\eta = 1 - F(T)$. We prove by contradiction. Suppose a nondegenerate combined contract $\{Y_0, Y_1, T\}$ with $x_B \in (0,1)$ is optimal. Since the buyer is indifferent between inspecting and not inspecting in equilibrium, we will have:

$$\begin{aligned} & (1-x_S)(R - Y_0 - Y_1 e^{-\alpha_B T}) - x_S(1-\mu)\left[Y_0 + Y_1 \eta e^{-\alpha_B T} + w\right] - I \\ = \; & (1-x_S)(R - Y_0 - Y_1 e^{-\alpha_B T}) - x_S\left[Y_0 + Y_1 \eta e^{-\alpha_B T} + w\right], \end{aligned}$$

which implies

$$x_S = \frac{1}{Y_0 + Y_1 \eta e^{-\alpha_B T} + w} \frac{I}{\mu}. \tag{3}$$

Then, the buyer's expected profit can be computed as follows:

$$\begin{aligned} \pi_B &= (1-x_S)(R - Y_0 - Y_1 e^{-\alpha_B T}) - x_S[Y_0 + Y_1 \eta e^{-\alpha_B T} + w] \\ &= (1-x_S)(R - Y_0 - Y_1 e^{-\alpha_B T}) - \frac{I}{\mu}. \end{aligned} \tag{4}$$

On the other hand, the inspection probability x_B is determined by the supplier's indifference condition:

$$Y_0 + Y_1 e^{-\alpha_S T} - qc_n = (1-x_B)(Y_0 + Y_1 \eta e^{-\alpha_S T}) + x_B(1-\mu)(Y_0 + Y_1 \eta e^{-\alpha_S T}) - qc_d$$

which leads to

$$x_B = \frac{q(c_n - c_d) - Y_1 e^{-\alpha_S T}(1-\eta)}{Y_0 + Y_1 e^{-\alpha_S T}\eta} \frac{1}{\mu}. \tag{5}$$

Now, consider a slight modification of the payment terms (Y_0, Y_1) that keeps $Y_0 + Y_1 e^{-\alpha_B T}$ constant (because of the discrete nature of the optimal sampling size problem, we could assume I and μ do not change as the result of the slight modification of the contract). Specifically, let $Y_0' = Y_0 + \epsilon$ and $Y_1' = Y_1 - \epsilon e^{\alpha_B T}$ where $\epsilon > 0$ is an infinitesimal amount. By continuity, the adulteration probability and the inspection probability are still in the interval $(0,1)$. Because x_S

decreases after the modification while $R - Y_0 - Y_1 e^{-\alpha_B T}$ stays unchanged, the buyer's profit π_B increases as a result of the modification. Moreover, the supplier's expected profit also strictly increases because

$$Y_0' + Y_1' e^{-\alpha_S T} = Y_0 + Y_1 e^{-\alpha_S T} + \epsilon(1 - e^{-\alpha T}) > Y_0 + Y_1 e^{-\alpha_S T}$$

where $\alpha = \alpha_S - \alpha_B > 0$. Hence, both the buyer and the supplier are better off with the modified contract. Clearly, the modified contract dominates the original one. Contradiction.

Note that if it is already that $x_B = 1$, then the modification described above may not be feasible because the inspection probability is bounded by 1. ■

Proof of Proposition 5: Again, for convenience, let $R = qr$, $w = qv_B - qr$, and $\eta = 1 - F(T)$. Consider a nondegenerate combined contract (Y_0, Y_1, T) that is optimal. By Proposition 4, we must have $x_B = 1$.

From the proof of Proposition 4, the requirement that $x_B \leq 1$ is equivalent to the following condition:

$$Y_0 + Y_1 e^{-\alpha_S T}\gamma \geq \frac{q(c_n - c_d)}{\mu}, \tag{6}$$

where $\gamma = \eta + \frac{1-\eta}{\mu} > 1$.

Consider again the modification in the proof of Proposition 4. If $e^{-\alpha T}\gamma \leq 1$, then the left-hand-side of (6) increases after the modification because

$$Y_0 + \epsilon + (Y_1 - \epsilon e^{\alpha_B T})e^{-\alpha_S T}\gamma = Y_0 + Y_1 e^{-\alpha_S T}\gamma + (1 - e^{-\alpha T}\gamma)\epsilon \geq Y_0 + Y_1 e^{-\alpha_S T}\gamma.$$

This implies that after the modification, x_B will decrease and become smaller than 1 in the new equilibrium, which asserts that the modification is still feasible. Hence, the original contract cannot be optimal. So, for a nondegenerate combined mechanism to be optimal, we must have $e^{-\alpha T}\gamma > 1$, or equivalently,

$$\mu < \frac{1 - \eta}{e^{\alpha T} - \eta}.$$

Because $\mu = 1 - (1 - \mu_0)^m$, the above condition is equivalent to

$$m < \frac{\ln(\frac{e^{\alpha T} - 1}{e^{\alpha T} - \eta})}{\ln(1 - \mu_0)}. \tag{7}$$

Given $\eta \geq 0$ and $T > L$, a necessary condition for the above inequality to hold is

$$m < \frac{\ln(1 - e^{-\alpha L})}{\ln(1 - \mu_0)}. \tag{8}$$

Furthermore, based on (3) and (4), the buyer's profit function can be written as

$$\pi_B = R - Y_0 - Y_1 e^{-\alpha_B T} - \frac{I}{\mu}\frac{R + w - Y_1(1 - \eta)e^{-\alpha_B T}}{Y_0 + Y_1 \eta e^{-\alpha_B T} + w}. \tag{9}$$

The IR constraint is

$$Y_0 + Y_1 e^{-\alpha_S T} \geq q c_n.$$

Because $x_B = 1$, we have $Y_0 + Y_1 e^{-\alpha_S T}\gamma = \frac{q(c_n - c_d)}{\mu}$. Hence, we can formulate the problem of finding the optimal nondegenerate combined contract as:

$$\begin{aligned} \max_{Y_0 \geq 0, Y_1 \geq 0, T > L} \; & \pi_B = R - Y_0 - Y_1 e^{-\alpha_B T} - \frac{I}{\mu} \frac{R + w - Y_1(1-\eta) e^{-\alpha_B T}}{Y_0 + Y_1 \eta e^{-\alpha_B T} + w} \\ s.t. \quad & Y_0 + Y_1 e^{-\alpha_S T}\gamma = \frac{q(c_n - c_d)}{\mu} \\ & Y_0 + Y_1 e^{-\alpha_S T} \geq q c_n. \end{aligned} \tag{10}$$

Clearly, for the feasible set to be nonempty, a necessary condition is

$$\frac{c_n - c_d}{\mu} > c_n \Rightarrow m < \frac{\ln(c_d/c_n)}{\ln(1-\mu_0)}. \tag{11}$$

Combining the inequalities in (8) and (11), we obtain the following necessary condition for a nondegenerate combined contract to be optimal:

$$m < \hat{m} = \min\left\{ \frac{\ln(c_d/c_n)}{\ln(1-\mu_0)}, \frac{\ln(1-e^{-\alpha L})}{\ln(1-\mu_0)} \right\}. \tag{12}$$

Because $m \geq 1$, (12) immediately implies the condition, which completes the proof for the upper bound of μ_0.

We now derive a lower bound of inspection precision μ_0 below which the combined contract is suboptimal. Recall the buyer's optimization problem of (10) and define $y = Y_1 e^{-\alpha_B T}$, $\beta = \frac{q(c_n - c_d)}{\mu}$, and $\xi = e^{-\alpha T}\gamma - 1$. Write Y_0 as $Y_0 = \beta - y e^{-\alpha T}\gamma$. Clearly, for $Y_0 \geq 0$, we need $y \leq \frac{\beta}{\gamma} e^{\alpha T}$. From the IR constraint $Y_0 + y e^{-\alpha T} \geq q c_n$, we have

$$y \leq \frac{\beta - q c_n}{\gamma - 1} e^{\alpha T}.$$

Thus, the buyer's optimization program can be reformulated as:

$$\begin{aligned} \max_{y, T, Y_0} \; & \pi_B(y, T, Y_0) = R - \beta + \xi y - \frac{I}{\mu} \frac{R + w - y(1-\eta)}{Y_0 + y\eta + w} \\ s.t. \quad & 0 \leq y \leq \min\{\frac{\beta - q c_n}{\gamma - 1} e^{\alpha T}, \frac{\beta}{\gamma} e^{\alpha T}\},\; Y_0 = \beta - y e^{-\alpha T}\gamma. \end{aligned} \tag{13}$$

Define ν as

$$\nu \equiv \min\left\{ (1-\theta)\frac{e^{\alpha T_a}}{F(T_a)}, e^{\alpha T_b}, 1 + (1-\theta)\frac{e^{\alpha T_c} - 1}{F(T_c)} \right\}.$$

Then the buyer's profit from the optimal deferred payment contract can be written as $R - q c_n \nu$. Note that because $\nu = (1-\theta)\frac{e^{\alpha T_a}}{F(T_a)}$ only when $\theta \leq 1 - F(T_a)$ (i.e., $T_a \leq T_b$), we always have $\nu > 1$.

Clearly, the combined contract is suboptimal if its profit is below $R - q c_n \nu$, that is,

$$q c_n \nu < \beta - \xi y + \frac{I}{\mu} \frac{R + w - y(1-\eta)}{Y_0 + y\eta + w}. \tag{14}$$

Because

$$\beta - \xi y > \beta - \frac{\xi\beta e^{\alpha T}}{\gamma} = \frac{e^{\alpha T}\beta}{\gamma} > 0,$$

a sufficient condition for (14) to hold is

$$qc_n\nu < \frac{I}{\mu}\frac{R + w - y(1-\eta)}{Y_0 + y\eta + w},$$

or equivalently,

$$\mu < \Big(\frac{I_0}{q} + I_c\Big)\frac{qv_B - y(1-\eta)}{qc_n\nu\big(v_B - r + \frac{Y_0+y\eta}{q}\big)}.$$

Because $Y_0 + y < qr$ (otherwise, the buyer will not participate) and $\eta \in (0,1)$, (14) holds if

$$\mu < \Big(\frac{I_0}{q} + I_c\Big)\frac{qv_B - qr}{qc_n\nu\big(v_B - r + \frac{qr}{q}\big)} = \Big(\frac{I_0}{q} + I_c\Big)\frac{v_B - r}{c_n\nu v_B}.$$

Therefore, a necessary condition for the combined contract to be optimal is to have

$$\mu > \Big(\frac{I_0}{q} + I_c\Big)\frac{v_B - r}{c_n\nu v_B}.$$

Using the fact that $\mu \le 1 - (1-\mu_0)^q$, we obtain a lower bound for the inspection precision μ_0 as

$$\mu_0 > 1 - \Big[1 - \Big(\frac{I_0}{q} + I_c\Big)\frac{v_B - r}{c_n\nu v_B}\Big]^{\frac{1}{q}}.$$

Notice that the lower bound is well-defined for reasonable inspection cost. For example, if the unit inspection cost $(I_0 q^{-1} + I_c)$ is lower than the unit production cost without adulteration (c_n), then

$$\Big(\frac{I_0}{q} + I_c\Big)\frac{v_B - r}{c_n\nu v_B} = \frac{I_0 q^{-1} + I_c}{c_n}\cdot\frac{1}{\nu}\cdot\frac{v_B - r}{v_B} < 1.$$

■

Online Appendix B: Algorithm for Solving Optimal Combined Contract

Below, we provide a procedure to solve for the optimal nondegenerate combined contract. From the proof of Proposition 5, the buyer's optimization problem can be rewritten as

$$\max_{y,T} \pi_B(y,T) = R - \beta + \xi y - \frac{I}{\mu}\frac{R + w - y(1-\eta)}{\beta + w - y(1-\eta+\xi)} \tag{15}$$
$$s.t. \quad 0 \le y \le \min\{\frac{\beta - qc_n}{\gamma - 1}e^{\alpha T}, \frac{\beta}{\gamma}e^{\alpha T}\}.$$

Notice that

$$\frac{\partial \pi_B(y,T)}{\partial y} = \xi - \frac{I}{\mu}\frac{(R-\beta)(1-\eta) + (R+w)\xi}{(\beta + w - y(1-\eta+\xi))^2} \text{ and } \beta + w - y(1-\eta+\xi) = Y_0 + y\eta + w > 0.$$

we have

$$\frac{\partial^2 \pi_B(y,T)}{\partial y^2} < 0.$$

Thus, for a combined contract to be optimal, y must satisfy

$$y^*(T) = \max\left\{0, \min\left\{\frac{1}{1-\eta+\xi}\left(\beta + w - \sqrt{\frac{(R-\beta)(1-\eta)+(R+w)\xi}{\mu\xi}\cdot I}\right), \frac{\beta - qc_n}{\gamma-1}e^{\alpha T}, \frac{\beta}{\gamma}e^{\alpha T}\right\}\right\}. \tag{16}$$

Determining the sampling size m^* in equilibrium is analogous to that in the pure inspection game. The buyer's profit is

$$(1-x_S)(R - Y_0 - Y_1 e^{-\alpha_B T}) - x_S(1-\mu)\left[Y_0 + Y_1\eta e^{-\alpha_B T} + w\right] - I.$$

In the second stage, the buyer determines the optimal m by maximizing

$$-x_S(1-\mu_0)^m\left(Y_0 + Y_1\eta e^{-\alpha_B T} + w\right) - mI_c.$$

The first order condition implies

$$-x_S(1-\mu_0)^m \ln(1-\mu_0)\left(Y_0 + Y_1\eta e^{-\alpha_B T} + w\right) - I_c = 0.$$

Because in equilibrium

$$x_S = \frac{1}{Y_0 + Y_1\eta e^{-\alpha_B T} + w}\frac{I}{\mu},$$

the first order condition could be written as

$$-(I_0 + mI_c)\ln(1-\mu_0)\frac{(1-\mu_0)^m}{1-(1-\mu_0)^m} = I_c,$$

or equivalently,

$$(1-\mu_0)^{-m} + I_c^{-1}(I_0 + I_c m)\ln(1-\mu_0) = 1.$$

From the pure inspection mechanism, we know the unique solution to the above equation is $\bar{m}$. Hence, the equilibrium sampling size is $m^* = \min\{\bar{m}, q\}$.

The remaining issue is to determine the optimal T for the objective function. From (7) and using the fact that $m \geq 1$, the optimal T must satisfy

$$\frac{\ln\left(\frac{e^{\alpha T}-1}{e^{\alpha T}-\eta}\right)}{\ln(1-\mu_0)} \geq 1,$$

which is equivalent to

$$T \leq \frac{1}{\alpha}\ln\left(\frac{1}{\mu_0} - \eta(\mu_0^{-1} - 1)\right).$$

Furthermore, since $T > L$ and $\eta > 0$, the optimal T must satisfy:

$$L < T < \bar{T} = -\frac{\ln \mu_0}{\alpha}. \tag{17}$$

Finally, because the smallest unit of T is one day for a realistic problem, T can be chosen from a finite set of values, i.e., $T = k/365$, $k = 1, 2, \cdots, \overline{k}$, where $\overline{k}$ is the largest integer such that $\overline{k}/365 \leq \bar{T}$. Based on (15), (16) and (17), we can use the following algorithm to find the optimal nondegenerate combined contract.

Algorithm 1 Algorithm to Find the Optimal Nondegenerate Combined Contract (Y_0, Y_1, T, m)

Require: (12) to be satisfied

$k = 1, t = 1/365, T = L$

while $T < \bar{T}$ **do**

$T = T + t, m = \min\{q, \bar{m}\}$

Compute y using (16) and compute the buyer's expected profit π_B corresponding to (y, T, m) using (15). Store (y, T, m, π_B).

end while

Compare profits and pick the optimal nondegenerate combined contract.

Compute Y_0, Y_1 and output (Y_0, Y_1, T, m).