Optimal Design of Deferred Payment Contract for Quality Control

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Outline

Quality management

Quality management philosophies

- "Quality means doing it right when no one is looking" – Henry Ford
- "Building a culture of stopping to fix problems, to get quality right the first time" – Principle of The Toyota Way

New challenges as supply chains become longer and global

Recent recalls

- Graco crib (LaJobi) recall (2010)
- Britax Chaperone car seat (Chinese supplier) recall (2010)
- GE coffee maker (Chinese supplier) recall (2010)

Figure 1: Source: http://www.cpsc.gov

New challenges as supply chains become longer and global

- Some of the quality problems are due to design problems
- Some are due to supplier adulteration
  - Not rigidly follow required manufacturing processes
  - Reduce or use cheap materials
  - Lack of rigorous quality control

Quality control and recent research

- Certification – Preproduction (ISO9000): Hwang et al. (2006)
- Inspection – Postproduction & Presales: Baiman et al. (2000, 2001), Balachandran and Radhakrishnan (2005)
- Liability and Warranty – Postsales: Revniers and Tapiero (1995), Lim (2001), Chao et al. (2009)
- Deferred Payment – Postsales: Babich and Tang (2012)

Deferred payment

"Protect Yourself When Outsourcing to China", WSJ 8/18/2013

"... structuring payments according to performance. Make sure your contract allows you to reserve the right to pay less or impose a penalty if a batch doesn't meet your expectations ..." [a]

[a] http://online.wsj.com/article/SB10001424127887323681904578639461757495312.html#articleTabs%3Darticle

Deferred Payment

Provide a proportion of the total payment to the supplier at the delivery; Withhold another proportion for a pre-specified amount of time and will release it contingent on no-complaint and quality problems discovered till that time.

Research Question

How to optimally design the deferred payment mechanisms?

- Should we defer all payment to a future date or should we defer part of the payment to a future date while paying the rest upfront ?
- What is the optimal deferral period?
- If partial deferral is optimal, what proportion should we defer?
- How does deferred payment mechanism compare with inspection mechanism? Are they complementary or substitutive?

Outline

Model Setup – Failure Process

- The buyer plans to procure q units from the supplier
- After accepting the contract, the supplier can choose whether to adulterate or not
  - Without adulteration, the products never fail in the lifecycle
  - With adulteration, each of the products fails in a stochastic amount of time τ_i where $\tau_i = L + z_i$.
  - L captures the latency between the moment the supplier incurs the financial cost of manufacturing the product and the earliest possible moment at which product failure is reported by some customer. L can be the lead time or some strictly positive time for certain product to reveal defection.
  - z_i i.i.d. from $F_0(\cdot)$ which has a positive dentisity function $f_0(\cdot)$.

Model Setup – Agents and Contract

Supplier

- Unit cost with adulteration: c_d
- Unit cost w/o adulteration: c_n ($> c_d$)
- Interest rate: α_S

Buyer

- Interest rate: α_B
- PV revenue: r
- PV liability: v_B ($> r$)

Contract (q, Y_0, Y_1, T)

- q: procurement quantity
- Y_0: initial payment at shipping/delivery
- Y_1: deferred payment contingent on no customer failure report up to T
- T: deferred duration

Outline

A First Attack

Proposition 1

The optimal deferred payment contract $\{Y_0^*, Y^*, T^*\}$ for the buyer can only be one of the following:

a) $Y_0 = 0$, $Y = \frac{q(c_n - c_d)}{F(T_a)}$, $T = T_a$, and the buyer's profit is $\pi_B^D(Y_0, Y, T) = qr - q(c_n - c_d)\frac{e^{\alpha T_a}}{F(T_a)}$, where $T_a = \arg\min_T \frac{e^{\alpha T}}{F(T)}$;

b) $Y_0 = 0$, $Y = qc_n$, $T = T_b$, and the buyer's profit is $\pi_B^D(Y_0, Y, T) = qr - qc_n e^{\alpha T_b}$, where T_b solves $F(T) = 1 - \frac{c_d}{c_n}$;

c) $Y_0 = qc_n - \frac{q(c_n - c_d)}{F(T_c)}$, $Y = \frac{q(c_n - c_d)}{F(T_c)}$, $T = T_c$, and the buyer's profit is $\pi_B^D(Y_0, Y, T) = qr - qc_n - q\left(c_n - c_d\right)\frac{e^{\alpha T_c} - 1}{F(T_c)}$, where $T_c = \arg\min_T \frac{e^{\alpha T} - 1}{F(T)}$. The supplier obtains $\pi_S^D(Y_0, Y, T) = \frac{q(c_n - c_d)}{F(T_a)} - qc_n$ in a) and zero in both b) and c).

A First Attack

Proposition 2

The optimal deferred payment contract takes the form of Proposition 1(a) if $T_a < T_b$; it takes the form of Proposition 1(c) if $T_a > T_b$ and $\frac{e^{\alpha T_b}-1}{e^{\alpha T_c}-1} > \frac{F(T_b)}{F(T_c)}$; otherwise, it takes the form of Proposition 1(b).

Discounted Unimodality

Definition

Given a discount ratio $\alpha \in (0, 1)$, a cumulative distribution function $F(t)$ with the support $[0, \infty)$ is said to satisfy the discounted unimodal property if the following function is unimodal.

$$\frac{F(t)}{e^{\alpha t} - 1}$$

Examples

- Weibull distribution (exponential distribution is a special case)
- Gompertz distribution
- Log Normal distribution
- Gamma distribution
- ...

Moral Hazard Condition & Information Accumulation Condition

Proposition 3

If $F(t)$ satisfies the discounted unimodal property, then a positive upfront payment is optimal if and only if the following two conditions are satisfied:

- moral hazard condition: $\theta \equiv c_d/c_n > 1 - F(T_a)$, that is, the moral hazard problem is not too severe;
- information accumulation condition: $\frac{d}{dt}\big(\ln F(t)\big)\big|_{t=T_b} > \frac{d}{dt}\big(\ln(e^{\alpha t} - 1)\big)\big|_{t=T_b}$, that is, at the optimal deferral time without upfront payment, the instantaneous speed at which the information accumulation rate grows exceeds the instantaneous speed at which the interest penalty grows.

Example 1

Suppose $L = 0$, and the defect discovery process follows Weibull distribution, that is, $F_0(t) = 1 - e^{-(\tilde{\lambda} t)^k}$. Then $F(t) = 1 - e^{-(\lambda t)^k}$ and $f(t) = k\lambda(\lambda t)^{k-1} e^{-(\lambda t)^k}$ where $\lambda = \tilde{\lambda} q^{\frac{1}{k}}$. It is easy to check that $T_b = \lambda^{-1}(-\ln\theta)^{1/k}$, and the information accumulation condition can be written as

$$\frac{\theta}{1-\theta} \cdot k\lambda \cdot (-\ln\theta)^{\frac{k-1}{k}} \cdot \left(1 - e^{-\alpha \frac{(-\ln\theta)^{1/k}}{\lambda}}\right) > \alpha.$$

For k large enough, the information accumulation condition will be satisfied even if $q = 1$. On the other hand, it can be shown that the moral hazard condition is trivially satisfied when k goes to infinity. Therefore, with k large enough, partial deferral can be optimal for any $q \geq 1$ and $L \geq 0$.

Outline

Comparative Statics – Contract Structure

Proposition 4

Suppose $F(t)$ satisfies the discounted unimodal property. If a positive upfront payment is optimal when $L = L_1$, then a positive upfront payment is also optimal for $L > L_1$.

Proposition 5

Suppose $F(t)$ satisfies the discounted unimodal property and the hazard rate function $f_0(t)/(1 - F_0(t))$ is nonincreasing. If a positive upfront payment is optimal for quantity q_1, then a positive upfront payment is also optimal for $q_2 > q_1$.

Comparative Statics – Contract Structure

Example 2

We keep using the setting in Example 1 The hazard rate function is $H(t) = k\lambda(\lambda t)^{k-1}$, which is increasing in t. Note that $\lambda = \tilde{\lambda} q^{\frac{1}{k}}$ where $\tilde{\lambda}$ is the scale parameter of Weibull distribution when $q = 1$. Hence, an increase of q is equivalent to an increase of λ. It can be shown that a larger q always makes it easier to satisfy both the information accumulation condition and the moral hazard condition. Therefore, even though the hazard rate function of Weibull distribution is increasing, the conclusion in Proposition 5 still holds.

Comparative Statics – Contract Structure

Example 3

Let the defection discovery time follows Gompertz distribution with parameters $a > 0$, $b > 0$. The density function, cumulative distribution function, and the hazard rate function are the followings:

$$f_0(t) = ae^{bt}e^{-\frac{a}{b}(e^{bt}-1)}, F_0(t) = 1 - e^{-\frac{a}{b}(e^{bt}-1)}, H_0(t) = ae^{bt}.$$

Apparently, the hazard rate function is increasing in time. It turns out that the conclusion in Proposition 5 does not hold for any parameter $a > 0$ and $b > 0$. In fact, if the upfront payment is zero when $q = q_1$, then the upfront payment is always zero for any $q > q_1$.

Comparative Statics – Payment Terms and Deferral Time

Proposition 6

Suppose the optimal deferred payment contract follows case (a) of Proposition 1

- An increase of the moral hazard severity (i.e., a decrease of θ) leads to an increase of the deferred payment but has no effect on the optimal deferral time T_a;
- An increase of the lead time L leads to an increase of the optimal deferral time T_a but has no effect on the deferred payment;
- If the hazard rate function $f_0(t)/(1 - F_0(t))$ is nonincreasing, then, an increase of the procurement quantity q leads to a decrease of the optimal deferral time T_a and a decrease of deferred payment per unit.

Comparative Statics – Payment Terms and Deferral Time

Proposition 7

Suppose the optimal deferred payment contract follows case (b) of Proposition 1.

- An increase of the moral hazard severity (i.e., a decrease of θ) leads to an increase of the optimal deferral time T_b;
- An increase of the lead time L leads to an increase of T_b;
- An increase of the procurement quantity q leads to a decrease of T_b.

Comparative Statics – Payment Terms and Deferral Time

Proposition 8

Suppose the optimal deferred payment contract follows case (c) of Proposition 1.

- An increase of the moral hazard severity (i.e., a decrease of θ) leads to an increase of the deferred payment but has no effect on the optimal deferral time T_c;
- If $F(t)$ satisfies the discounted unimodal property, then
  - i) an increase of the lead time L leads to an increase of the optimal deferral time T_c, a decrease of the deferred payment, and an increase of the upfront payment;
  - ii) and further, if the hazard rate function $f_0(t)/(1 - F_0(t))$ is nonincreasing, an increase of the procurement quantity q leads to a decrease of the optimal deferral time T_c, a decrease of the deferred payment per unit, and an increase of the upfront payment per unit.

Outline

Characterization of the Optimal Contract

Proposition 9

Let $\theta \equiv \frac{c_d}{c_n}$. If $F_0(t) = 1 - e^{-\lambda t}$, then the optimal deferred payment contract takes the form of

- Proposition 1(a) if $\theta < \underline{\theta}$,
- Proposition 1(b) if $\underline{\theta} \leq \theta \leq \overline{\theta}$,
- Proposition 1(c) if $\theta > \overline{\theta}$,

where $\underline{\theta} \equiv \frac{\alpha}{q\lambda+\alpha}$ and $\overline{\theta}$ is the unique solution in the interval (0,1) of the following equation:

$$\frac{1-\theta}{\theta} = \frac{q\lambda}{\alpha}\left(1 - e^{-\alpha L}\theta^{\frac{\alpha}{q\lambda}}\right).$$

Numerical Illustration

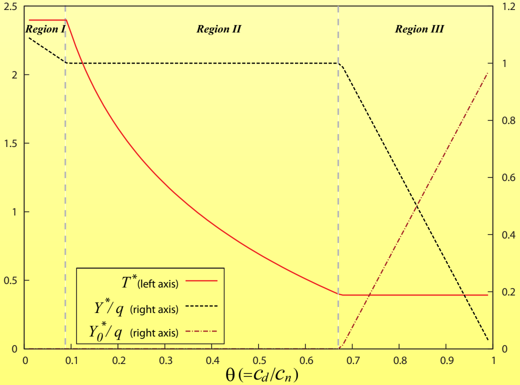

Figure 2: Demonstration of the Optimal Deferred Contract Structure

Numerical Illustration

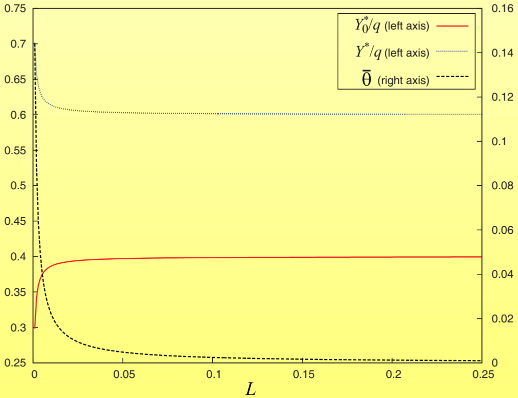

Figure 3: Demonstration of the normalized optimal payment amounts (Y_0^*/q, Y^*/q) and the threshold $\overline{\theta}$ as functions of the lead time L for the deferred payment mechanism. $F_0(t) = 1 - e^{-\lambda t}$ and the parameters are: $\lambda = 4$, $\alpha_S = 0.2$, $\alpha_B = 0.07$, $r = 5$, $c_n = 1$, $c_d = 0.4$, and $q = 1000$.

Numerical Illustration

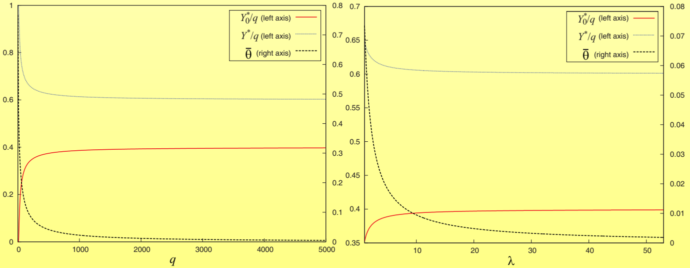

Figure 4: Demonstration of the normalized optimal payment amounts (Y_0^*/q, Y^*/q) and the threshold $\overline{\theta}$ as functions of the procurement quantity q (left plot) and the base discovery rate λ (right plot) for the deferred payment mechanism. $F_0(t) = 1 - e^{-\lambda t}$ and the parameters are: $\alpha_S = 0.2$, $\alpha_B = 0.07$, $r = 5$, $c_n = 1$, $c_d = 0.4$, $L = 0.01$, $\lambda = 4$ in the left plot, and $q = 1000$ in the right plot.

Outline

Model Setup – Inspection

Supplier

- Unit cost with adulteration: c_d
- Unit cost w/o adulteration: c_n ($> c_d$)

Buyer

- Interest rate: α_B
- PV revenue: r
- PV liability: v_B ($> r$)
- Cost of inspection is $I_0 + mI_c$ where m is sampling size the buyer chooses
- Inspection accuracy is μ_0 (no type I error).

Contract (q, Y_0)

- q: procurement quantity
- Y_0: payment contingent on products passing inspection

Timeline

0	$t < 0$	Buyer chooses deferred payment, inspection, or both, then offers the contract.
1	$t < 0$	Supplier accepts/rejects the contract. We assume the contract is accepted.
2	$t = 0$	Products are manufactured with/without adulteration by the supplier.
	$t = 0$	With inspection term, buyer decides whether to inspect.
	$t = 0$	Unless inspected and failed, products are shipped and/or delivered.
	$t = 0$	Initial payment is released.
3	$t = L$	Products have been delivered, distributed and sold. Product failure may be reported anytime from now on.
4	$t = T$	For deferred payment, if there is no failure report so far, the buyer releases the contingent payment.

Analysis of deferred payment

Buyer's Problem

- Let $\tau = \min\{\tau_1, \tau_2, \cdots, \tau_q\}$ and $F(t) = \Pr(\tau \leq t) = [1 - (1 - F_0(t - L))^q]\mathbf{1}_{t \geq L}$.
- The buyer optimizes her contract by:

$$\max_{Y_0 \geq 0, Y_1 > 0, T > L} qr - Y_0 - Y_1 e^{-\alpha_B T}$$

$$s.t. \quad Y_0 + Y_1 e^{-\alpha_S T} - qc_n \geq Y_0 + Y_1 e^{-\alpha_S T}(1 - F(T)) - qc_d,$$

$$Y_0 + Y_1 e^{-\alpha_S T} - qc_n \geq 0,$$

- Let $Y = Y_1 e^{-\alpha_S T}$, the PV of deferred payment from the supplier's perspective.

Analysis of Inspection

Supplier

- Unit cost with adulteration: c_d
- Unit cost w/o adulteration: c_n ($> c_d$)

Buyer

- Interest rate: α_B
- PV revenue: r
- PV liability: v_B ($> r$)
- Cost of inspection is $I_0 + mI_c$ where m is sampling size the buyer chooses
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Contract (q, Y_0)

- q: procurement quantity
- Y_0: payment contingent on products passing inspection

Mixed Strategy

x_S is adulteration probability and x_B is inspection probability

Analysis of Inspection

Recall $\theta = c_d/c_n$.

Proposition 10 (Equilibrium)

Let $\bar{m}$ be the unique solution of the following equation:

$$(1-\mu_0)^{-m} + I_c^{-1}(I_0 + I_c m)\ln(1-\mu_0) = 1.$$

If $q > \underline{q} \equiv \frac{v_B(I_0+I_c\bar{m})}{\mu_0(c_n+v_B-r)^2}$, then the inspection game has a unique equilibrium in which the buyer conducts inspection with probability $x_B^* = \frac{1-\theta}{1-(1-\mu_0)^{m^*}}$ for a sample size $m^* = \min\{\bar{m}, q\}$ and makes a contingent payment $Y_0^* = qc_n$ to the supplier on the pass of inspection; the supplier produces adulterated products with probability $x_S^* = \frac{I_0+I_c m^*}{\left(1-(1-\mu_0)^{m^*}\right)(qc_n+qv_B-qr)}$. The buyer achieves an equilibrium profit $\pi_B^I(x_B^*, m^*, x_S^*) = qr - qc_n - \frac{v_B\left(I_0+I_c m^*\right)}{\left(1-(1-\mu_0)^{m^*}\right)(c_n+v_B-r)}$ and the supplier obtains $\pi_S^I(x_B^*, m^*, x_S^*) = 0$.

Comparison of the two mechanisms

- Define the buyer's percentage loss under the two mechanisms:

$$\delta_j = 1 - \frac{\pi_B^j}{qr - qc_n}, \forall j \in \{D, I\}.$$

- We measure how much the buyer loses compared to the first-best under each mechanism.
- The smaller the δ, the better.

Comparison of the two mechanisms

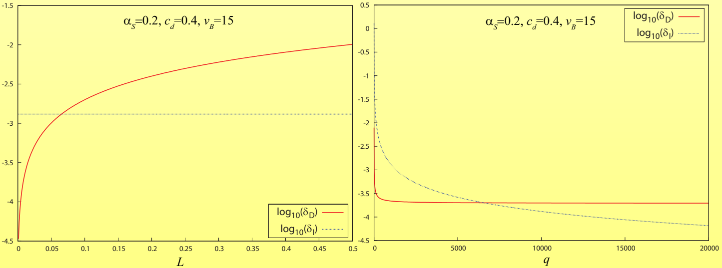

Figure 5: Demonstration of the (Log) efficiency loss ratios as functions of the lead time and the procurement quantity for the deferred payment and inspection mechanisms. $F_0(t) = 1 - e^{-\lambda t}$ and the other parameters are: $\lambda = 4$, $\alpha_B = 0.07$, $\alpha_S = 0.2$, $r = 5$, $v_B = 15$, $c_n = 1$, $c_d = 0.4$, $L = 0.01$, $I_0 = 3$, $I_c = 0.2$, and $\mu_0 = 0.6$.

The combined mechanism

Can we combine the two mechanisms to do better? Or is one mechanism sufficient?

Proposition 11

Any nondegenerate combined contract with $x_B \in (0, 1)$ is suboptimal.

Proposition 12

A nondegenerate combined contract (Y_0, Y_1, T) can be optimal only if

$$\mu_0 < \min\left\{e^{-\alpha L}, 1 - \frac{c_d}{c_n}\right\}$$

The combined mechanism

Analytically solving for the optimal combined mechanism is difficult. But we can solve it numerically with an efficient algorithm.

c_n	c_d	r	v_B	q	L
0.4	0.16	1	1.5	5	0.01
I_0	I_c	μ_0	λ	α_S	α_B
0.0005	0.0001	0.1	4	0.2	0.12
Y_0^*	Y_1^*	T^*	x_S^*	m^*	δ_B^{ID}
0.47061	1.5409	0.037397	0.00063304	5	0.0029735

Table 1: An example where the combined mechanism is optimal. δ_B^{ID} denotes the buyer's efficiency loss ratio under the optimal combined contract.

Conclusion

Contributions

- derive the optimal contract of the deferred payment mechanism under a general setting
- reveal the two economic driving forces that determines the optimal contract structure: the moral hazard severity and the information accumulation rate
- show that deferred payment mechanism can be complementary or substitutive to inspection mechanism and provide the necessary condition as well as a fast algorithm for the optimal combined mechanism

Thank you!