Optimal Incentives and Securitization of Defaultable Assets

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Outline of the Talk

Moral Hazard and Securitization: Empirical Evidence

- Mian and Sufi (2009)
- Downing, Jaffee, and Wallace (2009)
- Keys, Mukherjee, Seru, and Vig (2010)
- President Barack Obama (July 21, 2010)

Optimal Security Design: Theory

- Liquidy and Asymmetric Information
  - Meyers and Majluf (1984), Gorton and Pennachi (1990), Boot and Takor (1993), DeMarzo and Duffie (1999), Fulghieri and Lukin (2001), DeMarzo (2005), Axelson (2007)
- Dynamic moral hazard
  - Demarzo and Sannikov (2006), Cadenillas, Cvitanić and Zapatero (2007), Sannikov (2008)
- Persistent Moral Hazard
  - Hopenhayn and Jarque (2006), Hartman-Glaser, Piskorski, and Tchistyi (2009)

Outline of the Talk

The Model

- $t \in [0, \infty)$
- intermediary S can create a pool of N defaultable bonds (mortgages, loans, etc.)
- bonds pay a coupon u until default and a coupon $R < u$ after default
- the initial unobservable costly effort $e \in \{e_1, \cdots, e_K\}$ determines default risk
- defaults times in the pool are i.i.d. with a density $p_{e_j}(t)$ conditional on the effort e_j
- investors observe D_t, the number of defaults before time t

$$\tau_n = \inf\{t > 0 : D_t \geq n\}$$

The optimal contracting problem I.

- We distinguish two polar cases:
  - Competitive case: Intermediary designs the contract
  - Monopolistic Case: Investor designs the contract
-
  - **Conjecture 1.** Securitization leads to lax screening if the securitizer has all the bargaining power.
  - **Conjecture 2.** The optimal screening effort when the securitizer has all the bargaining power is lower than the optimal screening effort when the investor has all the bargaining power.

The optimal contracting problem II.

- a securitization contract specifies a transfer rates schedule $\{x_n(t,\tau_{[1,n]}),\ n\geq 0\}$ from the investor to the intermediary contingent on the history of defaults
- $$U_S(\{x_n\},e_j)\ \equiv\ E\left[\int_0^\infty e^{-\gamma\,t}\,u_S(x_{D_t}(t,\tau_{[1,D_t]}))\,dt\,|\,e_j\right]\ -\ C_j$$
- $$U_B(\{x_n\},e_j)\ \equiv\ E\left[\int_0^\infty e^{-r\,t}\,u_B(d_t\ -\ x_{D_t}(t,\tau_{[1,D_t]}))\,dt\,|\,e_j\right]\ .$$
- the risk sharing rule $J(x;d)$ solves

$$u_B'\big(d\ -\ w(J(x;d))\big)\,w'(J(x;d))\ =\ x$$

where $w(x)=u_S^{-1}(x)$.

Default times are order statistics

- The joint density of $(\tau_1, \cdots, \tau_k),\ k \le N$ conditional on the effort level e_j if given by

$$f_k^{e_j}(\tau_1, \cdots, \tau_k) \ = \ \mathbf{1}_{\tau_1 < \cdots < \tau_k} \frac{N!}{(N-k)!} p_{e_j}(\tau_1) \cdots p_{e_j}(\tau_k) \left(G_{e_j}(\tau_k)\right)^{N-k}$$

where

$$G_{e_j}(x) \ = \ \mathrm{Prob}[T_1 > t | e_j] \,.$$

- define

$$P_{k,e_i,e_j}(t;\tau_{[1,k]}) \ \equiv \ 1 \ - \ \frac{p_{e_i}(\tau_1) \cdots p_{e_i}(\tau_k)(G_{e_i}(t))^{N-k}}{p_{e_j}(\tau_1) \cdots p_{e_j}(\tau_k)(G_{e_j}(t))^{N-k}},$$

Outline of the Talk

The optimal contract

There exist Lagrange multipliers $\mu_{\mathrm{PC}},\ \mu_{\mathrm{IC}} \geq 0$ such that such that the optimal contract $\{x_n(t)\ ,\ n = 0, \cdots, N\}$ is given by

$$x_n(t, \tau_{[1,n]}) \ = \ \mathbf{1}_{\mathcal{I}}\, w \left(J \left(e^{(r-\gamma)t} \left(\mu_{\mathrm{PC}} \ + \ \sum_{i \neq j} \mu_{\mathrm{IC},i} P_{n,e_i,e_j}(t, \tau_{[1,n]}) \right) \ ; \, d_n \right)\right.$$

where

$$\mathcal{I} \ = \ \left\{ t \geq 0 \ : \ w \left(J \left(e^{(r-\gamma)t} \left(\mu_{\mathrm{PC}} \ + \ \sum_{i \neq j} \mu_{\mathrm{IC},i} P_{n,e_i,e_j}(t, \tau_{[1,k]}) \right) \ ; \, d_n \right)\right)\right)$$

If $\gamma > r$ then the contract has a finite maturity: There exists a $\bar{T} > 0$ such that $x_n(t, \tau_{[1,n]}) = 0$ for all $t \geq \bar{T}$ and all $n \geq 0$.

Properties of the optimal contract

- if optimal level of effort e_j implements the minimal default hazard rate then x_n are decreasing with n
- if optimal level of effort e_j does not implement the minimal default hazard rate and the recovery rate R/u is not too small then x_n increase in n for some values of $(t, \tau_{[1,n]})$;
- if the optimal level of effort is such that the individual default likelihood ratio $p_{e_j}(t)/p_{e_i}(t)$ is increasing (decreasing) in t then $x_n(t, \tau_{[1,n]})$ is increasing (decreasing) in $\tau_{[1,n]}$.

Small Risk Aversion: Extreme Punishment for defaults

Theorem Suppose that p_{e_j} has the smallest hazard rate. Then, suppose that both agents have exponential (CARA) preferences

$$u_S(x) \ = \ A_S^{-1}(1 - e^{-A_S x}) \ , \ u_B(x) \ = \ A_B^{-1}(1 - e^{-A_B x}) \, .$$

Then, when A_B, A_S are sufficiently small and A_B/A_S is not too large, $x_n \equiv 0$ for all $n \geq 1$. That is, the contract only makes payments until the first default occurs.

Regularity Assumption

Definition We will say the default time distributions are k-regular if the function $x_0(t)$ can have at most k local maxima in $t \in [0, \mathbb{R}_+)$

The risk neutral limit when effort reduces hazard rates

Theorem Suppose that default time distributions are k-regular, $\gamma > r$ and we are implementing the lowest hazard rate. Then, in the risk neutral limit, the optimal contract takes the following form:

- There exists a $\kappa \in \{0, \cdots, k-1\}$ and time instants $0 \leq t_0 < \cdots < t_\kappa < \infty$ and $y_i \in \mathbb{R}_+,\ i = 0, \cdots, \kappa$ such that the optimal contract transfers a lump sum of y_i at time t_i if no defaults occur until t_i. That is, the transfer process is given by

$$\sum_{i=0}^{\kappa} \mathbf{1}_{t=t_i}\ \mathbf{1}_{t_i<\tau_1}\, y_i\, .$$

Exponential Densities

Proposition Suppose that $p_{e_i}(t) = \lambda_i \, e^{-\lambda_i \, t}$ for all $i = 1, \cdots, K$ and $\lambda_1 > \cdots > \lambda_K$. Then, default time distributions are $(K-1)$-regular and therefore the convergence result holds with $k = K - 1$.

Outline of the Talk

1. Introduction
2. Setup
3. Optimal Contract
4. **Black and Cox**
   - Risk Neutral Limit in the Black and Cox setting
5. Securitization and effort choice
   - Securitization and effort choice: risk neutral limit

The Black and Cox (1976) default time distributions

- $$dX_t \;=\; X_t\,(\mu\,dt \,+\, \sigma\,dB_t)$$

- $$\text{Prob}[\tau^{X_B} < t] \;\equiv\; 1 \;-\; \Phi\left(\frac{mt+a}{\sigma\sqrt{t}}\right) \;+\; e^{\frac{-2ma}{\sigma^2}}\,\Phi\left(\frac{mt-a}{\sigma\sqrt{t}}\right)$$

- density
$$p^{a,m,\sigma}(t) \;=\; \frac{a}{\sqrt{2\pi}\,\sigma\,t^{3/2}}\,e^{-\frac{(mt+a)^2}{2\sigma^2 t}}$$

- $$m \;= \mu - 0.5\sigma^2 > 0\;,\; a \;=\; \log(X_0/X_B) \;>\; 0\,.$$

- $$\Phi(x) \;\equiv\; \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-y^2/2}dy$$

Black and Cox default distributions: properties

The density $p^{a,m,\sigma}$ is

(1) increasing in a, m and decreasing in σ in the sense of $\prec_{hr}$ order;

(2) increasing in a in the sense of the $\prec_{lr}$ order;

(3) **decreasing** in m with respect to the $\prec_{lr}$ order

(4) neither increasing nor decreasing in σ with respect to the $\prec_{lr}$ order.

Screening in the Black and Cox setting

Suppose that $p_{e_k} = p^{(a_k, m_k, \sigma_k)}$ for some $a_k, m_k, \sigma_k > 0$.

(1) Higher effort reduces default risk if and only if $\frac{a_j}{\sigma_j}$ and $\frac{m_j a_j}{\sigma_j^2}$ are monotone increasing in j.

(2) if $\frac{m_j}{\sigma_j} \geq \frac{m_i}{\sigma_i}$ for all i then x_n is decreasing with n. Furthermore, $x_n(t, \tau_{[1,n]})$ is increasing (decreasing) in $\tau_k, k = 1, \cdots, n$ when $\tau_k < \min_{i \neq j} \bar{t}_{i,j}$ $(\tau_k > \max_{i \neq j} \bar{t}_{i,j})$.

(3) if $\frac{m_j}{\sigma_j} \leq \frac{m_i}{\sigma_i}$ for all i then x_n is decreasing with $n \leq m$ when $\tau_m \leq \min_{i \neq j} \hat{t}_{i,j}$ but is increasing in $n \geq k$ when $\tau_k \geq \max_{i \neq j} \hat{t}_{ij}$. Furthermore, $x_n(t, \tau_{[1,n]})$ is monotone increasing in $\tau_k, k = 1, \cdots, n$.

Paying at time zero is optimal

Proposition In the Black and Cox setting, the transfer rate $x_0(t;\tau_{[1,n]}),\ t \geq 0$ always attains a local maximum at $t = 0$. In the binary effort case, it has at most one positive local maximum. Thus, there exist thresholds $0 \ \leq \theta_0(n,\tau_{[1,n]}) \ \leq \ \theta_1(n,\tau_{[1,n]}) < \ \theta_2(n,\tau_{[1,n]})$ such that $x_n(t;\tau_{[1,n]}) \ > \ 0$ if and only if

$$t \in \ [\tau_n, \theta_0(n,\tau_{[1,n]})) \ \cup \ (\theta_1(n,\tau_{[1,n]})\,,\, \theta_2(n,\tau_{[1,n]}))\,.$$

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Optimal contract

Define

$$\phi_1(t) \equiv \frac{(e^{(\gamma-r)t}-1)}{1-(G_{e_L}(t)/G_{e_H}(t))^N} \text{ and } t_1^* \equiv \arg\min_{t\geq 0} \phi_1(t)$$

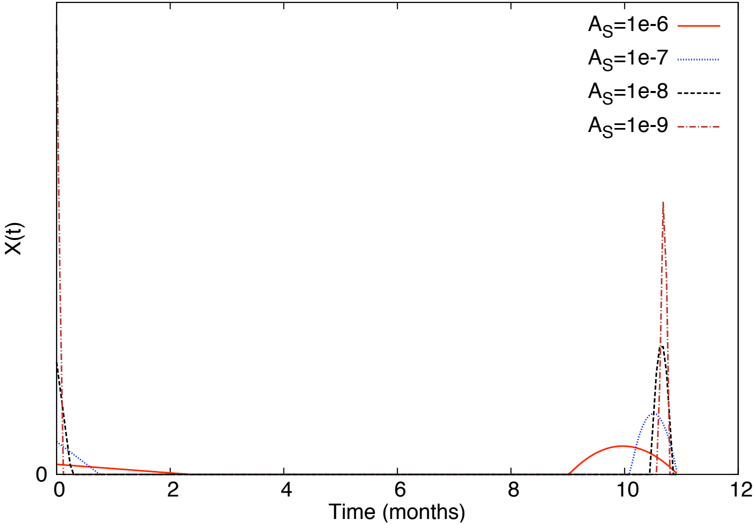
The following is true:

Theorem Suppose that the effort is binary, default time distributions are from the Black and Cox model, $p_{e_L} \prec_{hr} p_{e_H}$ and the desired effort level in e_H. Then, in the risk neural limit, the optimal contract makes a lump sum payment $y_0 \geq 0$ to the intermediary at time 0, and then a lump sum payment $y_1 > 0$ at a time $t^* > 0$ if no defaults occur before $t = t^*$. Furthermore, there exists a threshold C^* such that, for $C_H < C^*$ we have

$$t^* = t_1^* \,,\; y_1 = \frac{e^{\gamma t_1^*}(C_H - C_L)}{(G_{e_H}(t_1^*))^N - (G_{e_L}(t_1^*))^N},$$

and $y_0 > 0$.

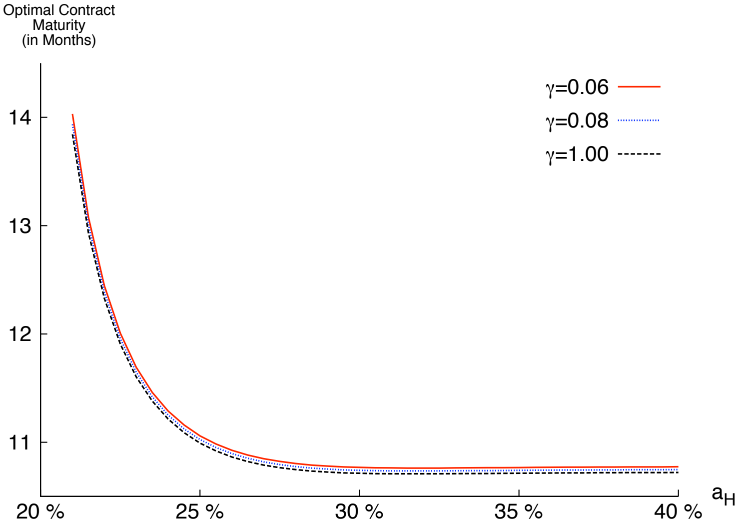
Figure: convergence



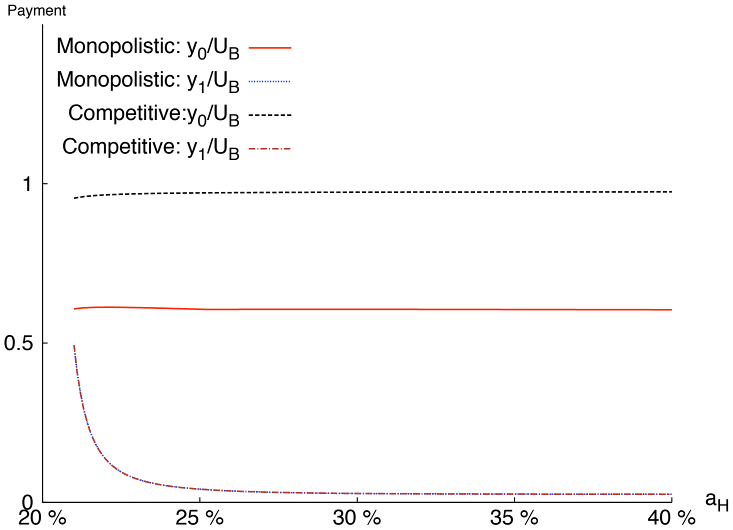
Optimal contract maturity

Proposition The maturity $t^* = t_1^*$ of the optimal contract is always monotone decreasing in N and $\gamma - r$ and is increasing in the size of default risk under high effort. The payment y_1 is increasing in $\gamma - r$, and decreasing in N and the size of default risk under high effort. Furthermore, t^* converges to 0 as $N \to \infty$.

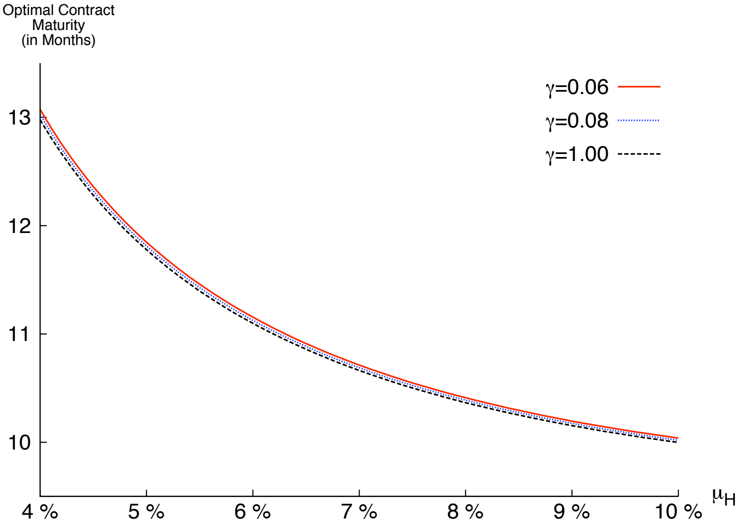
Maturity as a function of a_H



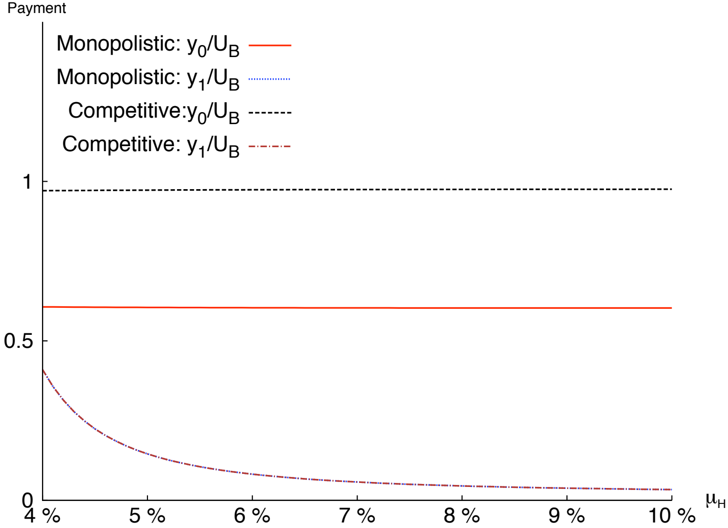
Payments as a function of a_H



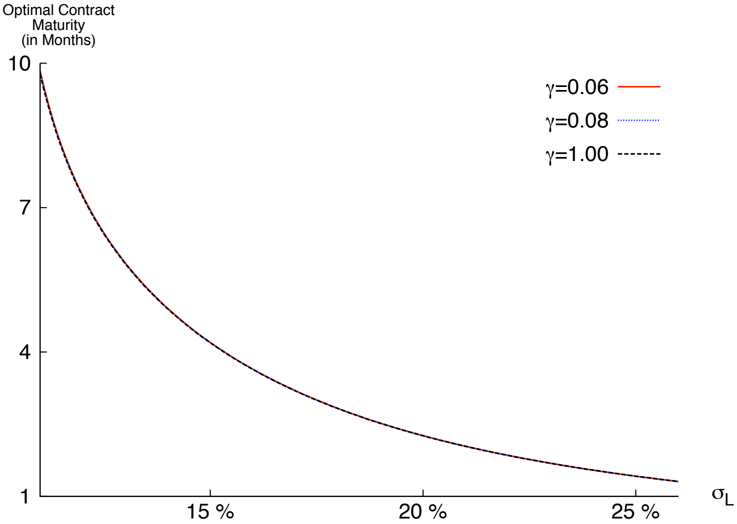
Maturity as a function of μ_H



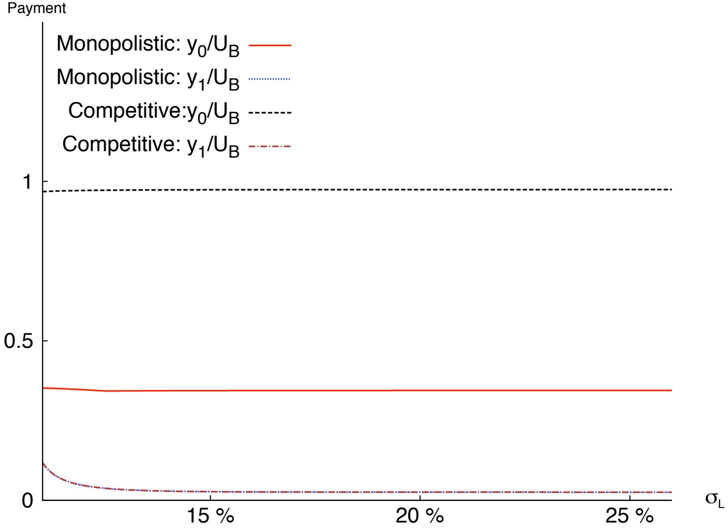
Payments as a function of μ_H



Maturity as a function of σ_L



Payments as a function of σ_L



Outline of the Talk

Outside options

- In order to address the equilibrium effort choice, we need to specify the outside options for the agents
- For the intermediary, we assume that the outside option is

$$U_S^0 = \max_j U_S(\{d_t\}, e_j)$$

- For the investor, we assume that the outside option is zero

The effect of risk aversion

Proposition Suppose that the intermediary has CARA preferences. Then, in the absence of securitization, the optimal effort level maximizing $U_S(\{d_n\}, e_j)$ is monotone decreasing in risk aversion and the discount rate γ.

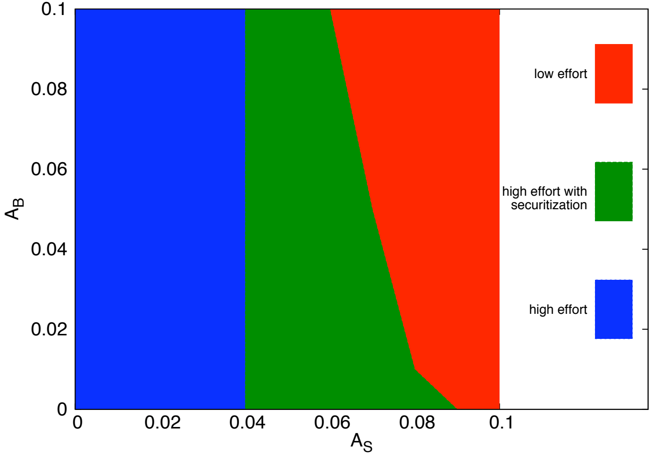
Retaining a fraction of the pool

Proposition Suppose that the intermediary has CARA preferences. Then the optimal effort level is monotone increasing in $\alpha \in [0, \min\{1, (A_S Nu)^{-1}\}]$ and is monotone **decreasing** in $\alpha \in [\min\{1, (A_S NR)^{-1}\}, 1]$.

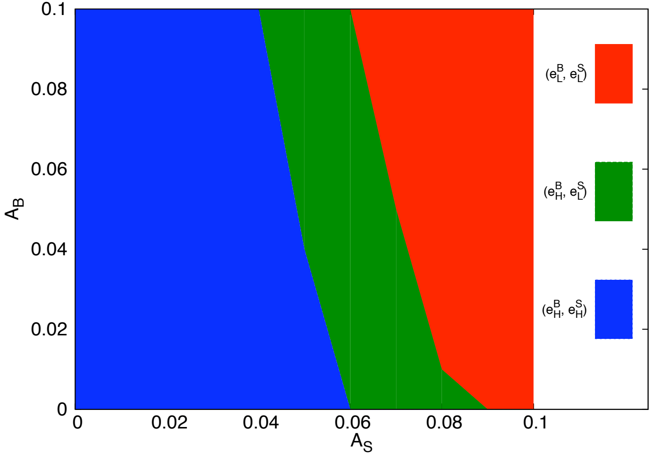
First loss piece

Proposition Suppose that the number N of assets in the pool is sufficiently large. Then, the optimal effort level is always monotone increasing in L for small positive values of L, but is monotone decreasing in L for large values of $L < N$.

Effort choice and optimal securitization



Effort choice and the bargaining power



Outline of the Talk

First best

- **Assumption** $C_j = N\,c_j\ ,\ j = H, L$ for some $c_L,\ c_H > 0$.
- **first best** surplus

$$\mathrm{FB}_j(N)\ =\ U_B(\{d_n\}; e_j) - U_B^0 - (U_S^0 + C_j)\ =\ N\ \cdot\ \mathrm{FB}_j(1)$$

is proportional to the number of assets in the pool.

Implementing low effort

In the risk neutral case, the optimal contract implementing low effort consists in paying the intermediary a fixed lump sum at time zero and the total surplus

- In the competitive case, this lump sum equals $U_B(\{d_n\}, e_L) - U_B^0$ (full surplus extraction by the intermediary).
- In the monopolistic case, the lump sum equals $U_S^0 + C_L$ (full surplus extraction by the investor).

In particular, total surplus coincides with FB_L.

Effort choice and menu

Proposition Suppose that the agents can only choose from a menu of contracts. Then, increasing the set of allowed contracts always (weakly) improves equilibrium effort level of the intermediary.

Second best surplus

Proposition The second best surplus SB_H for high effort is independent of bargaining power allocation and is given by

$$\mathrm{SB}_H(N) \;=\; N\,(\mathrm{FB}_H(1) - (c_H - c_L)\,\phi_1(t_1^*))\,.$$

The total surplus loss $(\mathrm{FB}_H(N) - \mathrm{SB}_H(N))/N$ per asset is monotone decreasing in the number N of assets and converges to zero as $N \to \infty$.

First best effort choice

- $\gamma > r$ implies

$$U_B(\{d_n\}, e_H) \; - \; U_B(\{d_n\}, e_L) \; > \; U_S(\{d_n\}, e_H) \; - \; U_S(\{d_n\}, e_L)\,.$$

- optimal effort level is high in the first best case if and only if

$$U_B(\{d_n\}, e_H) \; - \; U_B(\{d_n\}, e_L) \; > \; C_H - C_L\,. \qquad (1)$$

Second best effort choice

Proposition In the presence of securitization, equilibrium effort level is e_H if and only if

$$U_B(\{d_n\}, e_H) \; - \; U_B(\{d_n\}, e_L) \; \geq \; (1 + \phi_1(t_1^*))(C_H - C_L)\,.$$

Consequently,

- If (1) does not hold then the equilibrium effort level is e_L, both with and without securitization, independent of bargaining power allocation.
- If (1) holds then there exists an $N^* \geq 1$ such that the equilibrium effort level is e_H if and only if $N \geq N^*$. In particular, for sufficiently large N, securitization always improves equilibrium screening effort.

Securitization and the number of assets in the pool

Proposition Consider a 1-parameter family of distributions $G(t, \alpha)$, such that $G(t, \alpha)$ is continuous and increases in α in the hazard rate order. Suppose that $G_{e_j}(t) = G(t, \alpha_j),\ j = H, L$. Then, there exist thresholds $\underline{\alpha} < \overline{\alpha}$ such that:

- without securitization, intermediary chooses high effort if and only if $\alpha_H > \bar{\alpha}$;
- in the first best case, investor chooses high effort of the intermediary if and only if $\alpha_H > \underline{\alpha}$.

Then, for all $\alpha_H \in (\underline{\alpha}, \overline{\alpha})$, there exists a threshold $N^*(\alpha_H)$ such that equilibrium effort level is high if and only if $N > N^*(a_H)$. Consequently, for all $\alpha_H \in (\underline{\alpha}, \overline{\alpha})$ and $N > N^*(\alpha_H)$, securitization strictly improves equilibrium screening effort.

Figure: N^* as a function of a_H

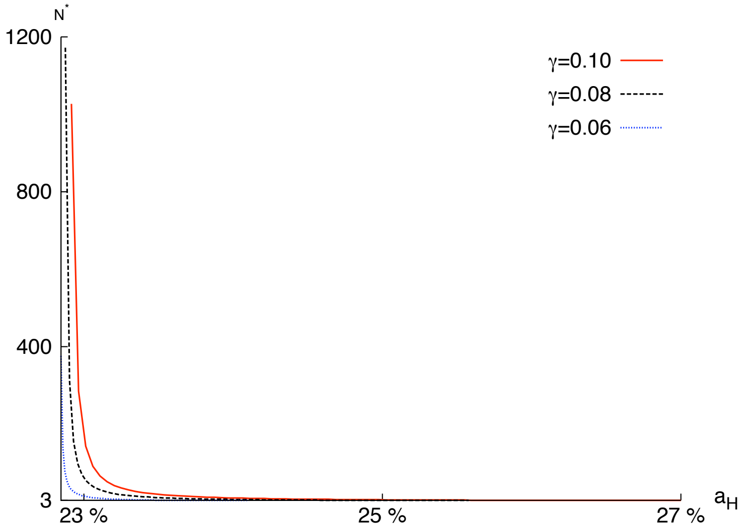
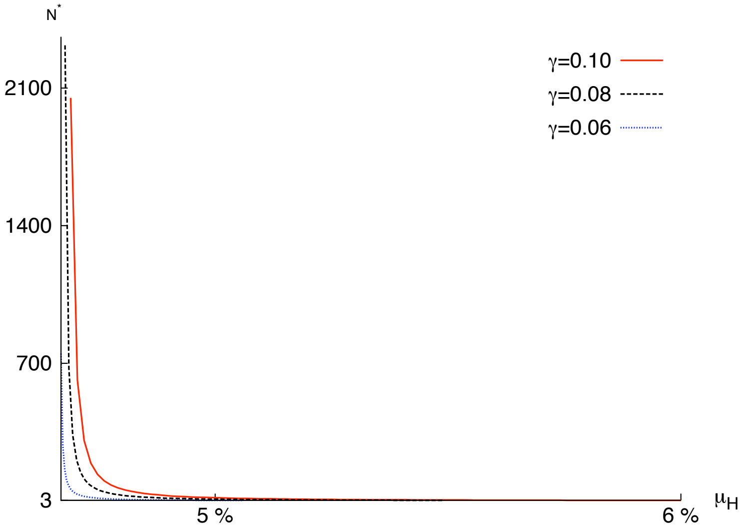


Figure: N^* as a function of μ_H



Thank You!